Indiana

DEPARTMENT OF EDUCATION

Dr. Jennifer McCormick

Superintendent of Public Instruction

Working Together for Student Success

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Indiana Academic Standards
Mathematics: Algebra I

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

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The Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Algebra I

Data Analysis and Statistics	
Guiding Principle:	
AI.DS.1	Understand statistics as a process for making inferences about a population based on a random sample from that population. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
AI.DS.2	Understand that statistics and data are non-neutral and designed to serve a particular interest. Analyze the possibilities for whose interest might be served and how the representations might be misleading.
AI.DS.3	Use technology to find a linear function that models a relationship between two quantitative variables to make predictions, and interpret the slope and y-intercept. Using technology, compute and interpret the correlation coefficient.
AI.DS.4	Describe the differences between correlation and causation.
AI.DS.5	Summarize bivariate categorical data in two-way frequency tables. Interpret relative frequencies in the contexts of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in data.

Number Systems and Expressions	
Guiding Principle:	
AI.NE.1	Explain the hierarchy and relationships of numbers and sets of numbers within the complex number system. Know that there is an imaginary number, *i*, such that $\sqrt{-1} = i$. Understand that the imaginary numbers along with the real numbers form the complex number system.

AI.NE.2	Simplify algebraic rational expressions, with numerators and denominators containing monomial bases with integer exponents, to equivalent forms.
AI.NE.3	Simplify square roots of monomial algebraic expressions, including non-perfect squares.
AI.NE.4	Factor quadratic expressions (including the difference of two squares, perfect square trinomials and other quadratic expressions).
AI.NE.5	Add, subtract, and multiply polynomials. Divide polynomials by monomials.

Functions	
Guiding Principle:	
AI.F.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Understand that if f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. Understand the graph of f is the graph of the equation y = f(x) with points of the form (x, f(x)).
AI.F.2	Evaluate functions for given elements of its domain, and interpret statements in function notation in terms of a context.
AI.F.3	Identify the domain and range of relations represented in tables, graphs, verbal descriptions, and equations.
AI.F.4	Describe, qualitatively, the functional relationship between two quantities by analyzing key features of a graph. Sketch a graph that exhibits given key features of a function that has been verbally described, including intercepts, where the function is increasing or decreasing, where the function is positive or negative, and any relative maximum or minimum

	values, Identify the independent and dependent variables.

Linear Equations, Inequalities, and Functions	
Guiding Principle:	
AI.L.1	Represent real-world problems using linear equations and inequalities in one variable, including those with rational number coefficients and variables on both sides of the equal sign. Solve them fluently, explaining the process used and justifying the choice of a solution method.
AI.L.2	Solve compound linear inequalities in one variable, and represent and interpret the solution on a number line. Write a compound linear inequality given its number line representation.
AI.L.3	Represent linear functions as graphs from equations (with and without technology), equations from graphs, and equations from tables and other given information (e.g., from a given point on a line and the slope of the line). Find the equation of a line, passing through a given point, that is parallel or perpendicular to a given line.
AI.L.4	Represent real-world problems that can be modeled with a linear function using equations, graphs, and tables; translate fluently among these representations, and interpret the slope and intercepts.
AI.L.5	Translate among equivalent forms of equations for linear functions, including slope-intercept, point-slope, and standard. Recognize that different forms reveal more or less information about a given situation.
AI.L.6	Represent real-world problems using linear inequalities in two variables and solve such problems; interpret the solution set and determine whether it is reasonable. Graph the solutions to a linear inequality in two variables as a half-plane.
AI.L.7	Solve linear and quadratic equations and formulas for a specified variable to highlight a quantity of interest, using the same reasoning as in solving equations.

Systems of Linear Equations and Inequalities	
Guiding Principle:	
AI.SEI.1	Understand the relationship between a solution of a system of two linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing; approximate solutions when the coordinates of the solution are non-integer numbers.
AI.SEI.2	Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions, including cases with no solution and infinitely many solutions. Solve systems of two linear equations algebraically using elimination and substitution methods.
AI.SEI.3	Write a system of two linear equations in two variables that represents a real-world problem and solve the problem with and without technology. Interpret the solution and determine whether the solution is reasonable.
AI.SEI.4	Represent real-world problems using a system of two linear inequalities in two variables. Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes with and without technology. Interpret the solution set and determine whether it is reasonable.

Quadratic and Exponential Equations and Functions	
Guiding Principle:	
AI.QE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. Understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Compare linear functions and exponential functions that model real-world situations using tables, graphs, and equations.

AI.QE.2	Represent real-world and other mathematical problems that can be modeled with simple exponential functions using tables, graphs, and equations of the form $y = ab^x$ (for integer values of $x > 1$, rational values of $b > 0$ and $b \neq 1$) with and without technology; interpret the values of a and b.
AI.QE.3	Use area models to develop the concept of completing the square to solve quadratic equations. Explore the relationship between completing the square and the quadratic formula.
AI.QE.4	Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), finding square roots, using the quadratic formula, and factoring, as appropriate to the initial form of the equation.
AI.QE.5	Represent real-world problems using quadratic equations in one or two variables and solve such problems with technology. Interpret the solution(s) and determine whether they are reasonable.
AI.QE.6	Graph exponential and quadratic functions with and without technology. Identify and describe key features, such as zeros, lines of symmetry, and extreme values in real-world and other mathematical problems involving quadratic functions with and without technology; interpret the results in the real-world contexts.
AI.QE.7	Describe the relationships among a solution of a quadratic equation, a zero of the function, an x-intercept of the graph, and the factors of the expression. Explain that every quadratic has two complex solutions, which may or may not be real solutions.

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Indiana Academic Standards
Mathematics: Algebra II

Introduction

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MATHEMATICS: Algebra II

Data Analysis, Statistics, and Probability	
Guiding Principle:	
AII.DSP.1	Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results
AII.DSP.2	Interpret and compare univariate data using measures of center (mean and median) and spread (range, inter-quartile range, standard deviation, and variance). Understand the effects of outliers on the statistical summary of the data.
AII.DSP.3	Use technology to find a linear, quadratic, or exponential function that models a relationship for a bivariate data set to make predictions; Interpret the correlation coefficient for linear models.
AII.DSP.4	Using the results of a simulation, decide if a specified model is consistent to those results. Construct a theoretical model and apply the law of large numbers to show the relationship between the two models.
AII.DSP.5	Understand dependent and independent events, and conditional probability; apply these concepts to calculate probabilities.
AII.DSP.6	Understand the Fundamental Counting Principle, permutations, and combinations; apply these concepts to calculate probabilities.

Complex Numbers and Expressions	
Guiding Principle:	
AII.CNE.1	Explain how extending the properties of integer exponents to rational numbers allows for a notation for radicals in terms of rational exponents (e.g. $5^{1/3}$) is defined to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must

	equal 5.)
AII.CNE.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
AII.CNE.3	Rewrite algebraic rational expressions in equivalent forms (e.g., using properties of exponents and factoring techniques). Add, subtract, multiply, and divide algebraic rational expressions.
AII.CNE.4	Rewrite rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).

Functions	
Guiding Principle:	
AII.F.1	Understand composition of functions and combine functions by composition.
AII.F.2	Define and find the inverse of a function. Verify functions are inverses algebraically and graphically.
AII.F.3	Understand that if the graph of a function contains a point (a, b), then the graph of the inverse relation of the function contains the point (b, a); the inverse is a reflection over the line y = x.
AII.F.4	Explore and describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative) with and without technology. Find the value of k given the graph of f(x) and the graph of f(x) + k, k f(x), f(kx), or f(x + k).

Systems of Equations and Inequalities
Guiding Principle:

AII.SEI.1	Solve a system of equations consisting of a linear equation and a quadratic equation in two variables algebraically and graphically with and without technology.
AII.SEI..2	Represent and solve real-world systems of linear equations and inequalities in two or three variables algebraically and using technology. Interpret the solution set and determine whether it is reasonable.
AII.SEI.3	Represent real-world problems using a system of linear equations in three variables. Understand that the algebraic steps to solve a two variable system can be extended to systems of equations in three variables.

Quadratic Equations and Functions	
Guiding Principle:	
AII.Q.1	Represent real-world problems that can be modeled with quadratic functions using tables, graphs, and equations; translate fluently among these representations. Solve such problems with and without technology. Interpret the solutions and determine whether they are reasonable.
AII.Q.2	Use completing the square to rewrite quadratic functions in vertex form and graph these functions with and without technology.
AII.Q.3	Understand that different forms of a quadratic equation can provide different information. Use and translate quadratic functions between standard, vertex, and intercept form to graph and identify key features, including intercepts, vertex, line of symmetry, end behavior, and domain and range.
AII.Q.4	Use the discriminant to determine the number and type of solutions of a quadratic equation. Find all solutions and write complex solutions in the form of a ± bi for real numbers a and b.

Exponential and Logarithmic Equations and Functions
Guiding Principle:

AII.EL.1	Graph exponential and logarithmic functions with and without technology. Identify and describe key features, such as intercepts, domain and range, asymptotes and end behavior. Know that the inverse of an exponential function is a logarithmic function.
AII.EL.2	Identify the percent rate of change in exponential functions. Classify them as representing exponential growth or decay.
AII.EL.3	Use the properties of exponents to rewrite expressions to describe transformations of exponential functions.
AII.EL.4	Use the properties of exponents to derive the properties of logarithms. Evaluate exponential and logarithmic expressions.
AII.EL.5	Solve exponential and logarithmic equations in one variable.
AII.EL.6	Represent real-world problems using exponential and logarithmic functions and solve such problems with technology. Interpret the solutions and determine whether they are reasonable.

Polynomial, Rational, and Other Equations and Functions	
Guiding Principle:	
AII.PR.1	Solve real-world and other mathematical problems involving polynomial equations with and without technology. Interpret the solutions and determine whether the solutions are reasonable.
AII.PR.2	Graph mathematical functions including: a. polynomial functions; b. rational functions; c. square root functions; d. absolute value functions; and, e. piecewise-defined functions with technology. Identify and describe features, such as intercepts, domain and range, end behavior, and lines of

	symmetry.
AII.PR.3	Solve real-world and other mathematical problems involving radical and rational equations. Give examples showing how extraneous solutions may arise.
AII.PR.4	Solve absolute value linear equations and inequalities in one variable.

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PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Analytical Algebra II

Data Analysis, Statistics, and Probability	
Guiding Principle: Data analysis, statistics, and probability content should be included throughout the course, as students collect and use univariate and bivariate data to create and interpret mathematical models. They should be able to make inferences and justify conclusions from various experimental and survey data, and develop a basic understanding of the structure of a good study, the biases that might exist, and the importance of randomization.	
AA.DSP.1	Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization and possible sources of bias relate to each.
AA.DSP.2	Choose, create, and critique, with technology, mathematical models (linear, quadratic and exponential) for bivariate data sets. Use the models to interpolate and/or extrapolate, to answer questions, and to draw conclusions or make decisions, addressing limitations and long-term ramifications. Recognize when a change in model is needed. Interpret the correlation coefficient for linear models.
AA.DSP.3	Read, interpret, and make decisions about data summarized numerically using measures of center and spread, in tables, and in graphical displays (line graphs, bar graphs, scatterplots, and histograms), e.g., explain why the mean may not represent a typical salary; critique a graphical display by recognizing that the choice of scale can distort information.
AA.DSP.4	Analyze and compare univariate data of two or more different data sets using measures of center (mean, median, and mode), shape, and spread (range, interquartile range, standard deviation, percentiles, and variance) making use of technology. Understand the effects of outliers on the statistical summary of the data.
AA.DSP.5	Record multiple observations (or simulated samples) of random events and construct empirical models of the probability distributions. Construct a theoretical model and apply the law of large numbers to show the relationship between the two models.

AA.DSP.6	Evaluate the validity of claims based on empirical probabilities and theoretical probabilities, including those derived from dependent and independent events. Draw conclusions and make decisions in various probabilistic contexts. Make use of different representations of data including two-way tables and tree diagrams.
AA.DSP.7	Determine the nature and number of elements in a finite sample space to model the outcomes of real-world events using the Fundamental Counting Principle, permutations, and combinations.

Linear Functions and Beyond	
Guiding Principle: Extending from work with linear functions in Algebra I, this content should include work with arithmetic sequences and series, understanding the relationship to linear functions. Additionally, students should solidify their understanding of systems of equations. The focus should be on solving systems of equations that represent real-world situations, with technology. Students should be able to solve systems that involve nonlinear equations. They should also be able to solve systems of equations with three variables with technology, using various strategies such as matrices.	
AA.LF.1	Model real world situations involving arithmetic sequences and understand that they can be defined both recursively and with an explicit formula.
AA.LF.2	Find partial sums of arithmetic series that model real world situations.
AA.LF.3	Recognize functional relationships in real world contexts. Translate fluently among multiple representations (graphs, tables, equations, and verbal descriptions).
AA.LF.4	Within real world contexts, understand composition of functions and combine functions by composition.
AA.LF.5	Explore and describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative) with and without technology. Find the value of k given the graph of f(x) and the graph of f(x) + k, k f(x), f(kx), or f(x + k).

AA.LF.6	Represent and solve real-world problems using a system of equations and/or inequalities consisting of a linear equation and a quadratic equation in two variables with technology.
AA.LF.7	Represent real-world problems using a system of linear equations and/or inequalities in two or three variables. Solve such systems graphically or with matrices, as appropriate to the system, with technology. Interpret the solution and determine whether it is reasonable.

Quadratic and Other Polynomial Functions	
Guiding Principle: Extending from Algebra I, students should be able to represent real-world problems that can be modeled with quadratic or higher-order polynomial functions, interpreting key attributes in a given context.	
AA.QP.1	Represent real-world problems that can be modeled with quadratic functions using tables, graphs, and equations; translate fluently among these representations. Solve such problems with technology. Interpret the solutions and determine whether they are reasonable.
AA.QP.2	Understand that different forms of a quadratic equation can provide different information. Identify and interpret within a given context the vertex, intercepts, zeros, domain and range, and lines of symmetry.
AA.QP.3	Represent real-world problems that can be modeled with polynomial functions using graphs and equations. Solve such problems with technology. Interpret the solutions and determine whether they are reasonable.
AA.QP.4	Graph polynomial functions that model a real-world situation with technology. Identify, describe, and interpret key features in the context of the situation, such as intercepts, zeros, domain and range, end behavior, maxima and minima, and lines of symmetry.

Exponential and Logarithmic Functions
Guiding Principle: Extending from initial work with exponential functions in Algebra I, students should understand the relationship between logarithmic and exponential functions. Additionally, this content should include representing real-world problems that can be modeled with either exponential or logarithmic functions, interpreting key attributes in a given context. Arithmetic and geometric sequences and series should also be introduced, making the connection to linear and exponential functions respectively.

AA.EL.1	Model real world situations involving geometric sequences and understand that they can be defined both recursively and with an explicit formula.
AA.EL.2	Find partial sums of geometric series that model real world situations.
AA.EL.3	Represent real-world problems using exponential functions in one or two variables and solve such problems with technology. Interpret the solutions and determine whether they are reasonable.
AA.EL.4	Graph exponential functions that model real-world situations with technology. Identify, describe, and interpret key features, such as intercepts, zeros, domain, range, asymptotic and end behavior.
AA.EL.5	Given real-world contexts, identify the percent rate of change in exponential functions. Classify them as representing exponential growth or decay.
AA.EL.6	Analyze growth and decay using absolute and relative change and make comparisons using absolute and relative difference.
AA.EL.7	Know that the inverse of an exponential function is a logarithmic function. Represent exponential and logarithmic functions that model real-world situations using graphing technology and describe their inverse relationship. Use the inverse relationship between exponential functions and logarithms to evaluate expressions and solve equations in one variable.

Rational, Radical, and Other Functions	
Guiding Principle: This content should include representing real-world problems that can be modeled with rational, radical, and piecewise-defined functions. Students should be able to translate between various representations and interpret key attributes in a given context.	
AA.R.1	Represent and solve real-world problems that can be modeled with rational functions using tables, graphs, and equations. Graph rational functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior.

AA.R.2	Represent and solve real-world problems that can be modeled with radical functions using tables, graphs, and equations. Graph radical functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior.
AA.R.3	Graph real-world functions including polynomial, rational, square root, step functions, absolute value functions, and piecewise-defined functions with technology. Identify and describe features, such as intercepts, domain and range, end behavior, asymptotic behavior, and/or lines of symmetry

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Indiana Academic Standards
Mathematics: Geometry

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Geometry

Logic and Proofs	
Guiding Principle:	
G.LP.1	Understand and describe the structure of and relationships within an axiomatic system (undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems). Understand the differences among supporting evidence, counterexamples, and actual proofs.
G.LP.2	Use precise definitions for angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, and plane. Use standard geometric notation.
G.LP.3	State, use, and examine the validity of the converse, inverse, and contrapositive of conditional ("if – then") and bi-conditional ("if and only if") statements.
G.LP.4	Understand that proof is the means used to demonstrate whether a statement is true or false mathematically. Develop geometric proofs, including those involving coordinate geometry, using two-column, paragraph, and flow chart formats.

Points, Lines, and Angles	
Guiding Principle:	
G.PL.1	Prove and apply theorems about lines and angles, including the following: ● Vertical angles are congruent. ● When a transversal crosses parallel lines, alternate interior angles are congruent, alternate exterior angles are congruent, and corresponding angles are congruent. ● When a transversal crosses parallel lines, same side interior angles are supplementary. ● Points on a perpendicular bisector of a line segment are exactly those equidistant from the endpoints of the segment.

G.PL.2	Explore the relationships of the slopes of parallel and perpendicular lines. Determine if a pair of lines are parallel, perpendicular, or neither by comparing the slopes in coordinate graphs and equations.
G.PL.3	Use tools to explain and justify the process to construct congruent segments and angles, angle bisectors, perpendicular bisectors, altitudes, medians, and parallel and perpendicular lines.
G.PL.4	Develop the distance formula using the Pythagorean Theorem. Find the lengths and midpoints of line segments in the two-dimensional coordinate system.

Triangles	
Guiding Principle:	
G.T.1	Prove and apply theorems about triangles, including the following: ● Measures of interior angles of a triangle sum to 180°. ● The Isosceles Triangle Theorem and its converse. ● The Pythagorean Theorem. ● The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length. ● A line parallel to one side of a triangle divides the other two proportionally, and its converse. ● The Angle Bisector Theorem.
G.T.2	Explore and explain how the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL) follow from the definition of congruence in terms of rigid motions.
G.T.3	Use tools to explain and justify the process to construct congruent triangles.
G.T.4	Use the definition of similarity in terms of similarity transformations, to determine if two given triangles are similar. Explore and develop the meaning of similarity for triangles.

G.T.5	Use congruent and similar triangles to solve real-world and mathematical problems involving sides, perimeters, and areas of triangles.
G.T.6	Prove and apply the inequality theorems, including the following: ● Triangle inequality. ● Inequality in one triangle. ● The hinge theorem and its converse.
G.T.7	Explore the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. Understand and use the geometric mean to solve for missing parts of triangles.
G.T.8	Compute perimeters and areas of polygons in the coordinate plane to solve real-world and other mathematical problems.
G.T.9	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.T.10	Use trigonometric ratios (sine, cosine,tangent and their inverses) and the Pythagorean Theorem to solve real-world and mathematical problems involving right triangles.
G.T.11	Explore the relationship between the sides of special right triangles (30° - 60° and 45° - 45°) and use them to solve real-world and other mathematical problems.

Quadrilaterals and Other Polygons	
Guiding Principle:	
G.QP.1	Prove and apply theorems about parallelograms, including those involving angles, diagonals, and sides.

G.QP.2	Prove that given quadrilaterals are parallelograms, rhombuses, rectangles, squares, kites, or trapezoids. Include coordinate proofs of quadrilaterals in the coordinate plane.
G.QP.3	Develop and use formulas to find measures of interior and exterior angles of polygons.
G.QP.4	Identify types of symmetry of polygons, including line, point, rotational, and self-congruences.
G.QP.5	Compute perimeters and areas of polygons in the coordinate plane to solve real-world and other mathematical problems.
G.QP.6	Develop and use formulas for areas of regular polygons.

Circles	
Guiding Principle:	
G.CI.1	Define, identify and use relationships among the following: radius, diameter, arc, measure of an arc, chord, secant, tangent, congruent circles, and concentric circles.
G.CI.2	Derive the fact that the length of the arc intercepted by an angle is proportional to the radius; derive the formula for the area of a sector.
G.CI.3	Explore and use relationships among inscribed angles, radii, and chords, including the following: ● The relationship that exists between central, inscribed, and circumscribed angles. ● Inscribed angles on a diameter are right angles. ● The radius of a circle is perpendicular to a tangent where the radius intersects the circle.
G.CI.4	Solve real-world and other mathematical problems that involve finding measures of circumference, areas of circles and sectors, and arc lengths and related angles (central, inscribed, and intersections of secants and tangents).

G.CI.5	Use tools to explain and justify the process to construct a circle that passes through three given points not on a line, a tangent line to a circle through a point on the circle, and a tangent line from a point outside a given circle to the circle.
G.CI.6	Use tools to construct the inscribed and circumscribed circles of a triangle. Prove properties of angles for a quadrilateral inscribed in a circle.

Transformations	
Guiding Principle:	
G.TR.1	Use geometric descriptions of rigid motions to transform figures and to predict and describe the results of translations, reflections and rotations on a given figure. Describe a motion or series of motions that will show two shapes are congruent.
G.TR.2	Verify experimentally the properties of dilations given by a center and a scale factor. Understand the dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Three-Dimensional Solids	
Guiding Principle:	
G.TS.1	Create a net for a given three-dimensional solid. Describe the three-dimensional solid that can be made from a given net (or pattern).
G.TS.2	Explore and use symmetries of three-dimensional solids to solve problems.
G.TS.3	Explore properties of congruent and similar solids, including prisms, regular pyramids, cylinders, cones, and spheres and use them to solve problems.

G.TS.4	Solve real-world and other mathematical problems involving volume and surface area of prisms, cylinders, cones, spheres, and pyramids, including problems that involve composite solids and algebraic expressions.
G.TS.5	Apply geometric methods to create and solve design problems.

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Indiana Academic Standards
Mathematics: Grade 1

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PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as

	digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.
PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS - Grade 1

The Mathematics standards for Grade 1 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Grade 1 are made up of five strands: Number Sense, Computation and Algebraic Thinking, Geometry, Measurement, and Data Analysis. The skills listed in each stand indicate what students in Grade 1 should know and be able to do in Mathematics.

NUMBER SENSE	
1.NS.1	Count to at least 120 by ones, fives, and tens from any given number. In this range, read and write numerals and represent a number of objects with a written numeral
1.NS.2	Understand that 10 can be thought of as a group of ten ones — called a “ten." Understand that the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. Understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NS.3	Match the ordinal numbers first, second, third, etc., with an ordered set up to 10 items.
1.NS.4	Use place value understanding to compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
1.NS.5	Find mentally ten more or ten less than a given two-digit number without having to count, and explain the thinking process used to get the answer.
1.NS.6	Show equivalent forms of whole numbers as groups of tens and ones, and understand that the individual digits of a two-digit number represent amounts of tens and ones.

COMPUTATION AND ALGEBRAIC THINKING	
1.CA.1	Demonstrate fluency with addition facts and the corresponding subtraction facts within 20. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a 10 (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). Understand the role of 0 in addition and subtraction.
1.CA.2	Solve real-world problems involving addition and subtraction within 20 in situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all parts of the addition or subtraction problem (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).
1.CA.3	Create a real-world problem to represent a given equation involving addition and subtraction within 20.
1.CA.4	Solve real-world problems that call for addition of three whole numbers whose sum is within 20 (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).
1.CA.5	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; describe the strategy and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and that sometimes it is necessary to compose a ten.
1.CA.6	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2).
1.CA.7	Create, extend, and give an appropriate rule for number patterns using addition within 100.

GEOMETRY	
1.G.1	Identify objects as two-dimensional or three-dimensional. Classify and sort two-dimensional and three-dimensional objects by shape, size, roundness and other attributes. Describe how two-dimensional shapes make up the faces of three-dimensional objects.
1.G.2	Distinguish between defining attributes of two- and three-dimensional shapes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size). Create and draw two-dimensional shapes with defining attributes.
1.G.3	Use two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. [In grade 1, students do not need to learn formal names such as "right rectangular prism."]
1.G.4	Partition circles and rectangles into two and four equal parts; describe the parts using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of, the parts. Understand for partitioning circles and rectangles into two and four equal parts that decomposing into equal parts creates smaller parts.

MEASUREMENT	
1.M.1	Use direct comparison or a nonstandard unit to compare and order objects according to length, area, capacity, weight, and temperature.
1.M.2	Tell and write time to the nearest half-hour and relate time to events (before/after, shorter/longer) using analog clocks. Understand how to read hours and minutes using digital clocks.
1.M.3	Identify the value of a penny, nickel, dime, and a collection of pennies, nickels, and dimes.

DATA ANALYSIS	
1.DA.1	Organize and interpret data with up to three choices (What is your favorite fruit? apples, bananas, oranges); ask and answer questions about the total number of data points, how many in each choice, and how many more or less in one choice compared to another.

Indiana
DEPARTMENT OF EDUCATION
Working Together for Student Success

Indiana Academic Standards
Mathematics: Grade 2

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create highest-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 2

The Mathematics standards for Grade 2 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Grade 2 are made up of five strands: Number Sense, Computation and Algebraic Thinking, Geometry, Measurement, and Data Analysis. The skills listed in each strand indicate what students in Grade 2 should know and be able to do in Mathematics.

NUMBER SENSE	
2.NS.1	Count by ones, twos, fives, tens, and hundreds up to at least 1,000 from any given number.
2.NS.2	Read and write whole numbers up to 1,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000.
2.NS.3	Plot and compare whole numbers up to 1,000 on a number line.
2.NS.4	Match the ordinal numbers first, second, third, etc., with an ordered set up to 30 items.
2.NS.5	Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by placing that number of objects in two groups of the same size and recognizing that for even numbers no object will be left over and for odd numbers one object will be left over, or by pairing objects or counting them by 2s).
2.NS.6	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand that 100 can be thought of as a group of ten tens - called a “hundred.” Understand that the numbers 100, 200, 200, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NS.7	Use place value understanding to compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

COMPUTATION AND ALGEBRAIC THINKING	
2.CA.1	Add and subtract fluently within 100.
2.CA.2	Solve real-world problems involving addition and subtraction within 100 in situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all parts of the addition or subtraction problem (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem). Use estimation to decide whether answers are reasonable in addition problems.
2.CA.3	Solve real-world problems involving addition and subtraction within 100 in situations involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers, and equations with a symbol for the unknown number to represent the problem).
2.CA.4	Add and subtract within 1000, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; describe the strategy and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones, and that sometimes it is necessary to compose or decompose tens or hundreds.
2.CA.5	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal groups.
2.CA.6	Show that the order in which two numbers are added (commutative property) and how the numbers are grouped in addition (associate property) will not change the sum. These properties can be used to show that numbers can be added in any order.
2.CA.7	Create, extend, and give an appropriate rule for number patterns using addition and subtraction within 1000.

GEOMETRY	
2.G.1	Identify, describe, and classify two- and three-dimensional shapes (triangle, square, rectangle, cube, right rectangular prism) according to the number and shape of faces and the number of sides and/or vertices. Draw two-dimensional shapes.
2.G.2	Create squares, rectangles, triangles, cubes, and right rectangular prisms using appropriate materials.
2.G.3	Investigate and predict the result of composing and decomposing two- and three-dimensional shapes.
2.G.4	Partition a rectangle into rows and columns of same-size (unit) squares and count to find the total number of same-size squares.
2.G.5	Partition circles and rectangles into two, three, or four equal parts; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, four fourths. Recognize that equal parts of identical wholes need not have the same shape.

MEASUREMENT	
2.M.1	Describe the relationships among inch, foot, and yard. Describe the relationship between centimeter and meter.
2.M.2	Estimate and measure the length of an object by selecting and using appropriate tools, such as rulers, yardsticks, meter sticks, and measuring tapes to the nearest inch, foot, yard, centimeter and meter.
2.M.3	Understand that the length of an object does not change regardless of the units used. Measure the length of an object twice using length units of different lengths for the two measurements. Describe how the two measurements relate to the size of the unit chosen.

2.M.4	Estimate and measure volume (capacity) using cups and pints.
2.M.5	Tell and write time to the nearest five minutes from analog clocks, using a.m. and p.m. Solve real-world problems involving addition and subtraction of time intervals on the hour or half hour.
2.M.6	Describe relationships of time, including: seconds in a minute; minutes in an hour; hours in a day; days in a week; and days, weeks, and months in a year.
2.M.7	Find the value of a collection of pennies, nickels, dimes, quarters and dollars.

DATA ANALYSIS	
2.DA.1	Draw a picture graph (with single-unit scale) and a bar graph (with single-unit scale) to represent a data set with up to four choices (What is your favorite color? red, blue, yellow, green). Solve simple put-together, take-apart, and compare problems using information presented in the graphs.

Indiana Academic Standards
Mathematics: Grade 3

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the highest-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

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PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 3

The Mathematics standards for Grade 3 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Grade 3 are made up of six strands: Number Sense, Computation, Algebraic Thinking, Geometry, Measurement, and Data Analysis. The skills listed in each strand indicate what students in Grade 3 should know and be able to do in Mathematics.

NUMBER SENSE	
3.NS.1	Read and write whole numbers up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 10,000.
3.NS.2	Compare two whole numbers up to 10,000 using >, =, and < symbols.
3.NS.3	Understand a fraction, 1/b, as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction, a/b, as the quantity formed by *a* parts of size 1/b. [*In grade 3, limit denominators of fractions to 2, 3, 4, 6, 8*.]
3.NS.4	Represent a fraction, 1/b, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
3.NS.5	Represent a fraction, a/b, on a number line by marking off lengths 1/b from 0. Recognize that the resulting interval has size a/b, and that its endpoint locates the number a/b on the number line.
3.NS.6	Understand two fractions as equivalent (equal) if they are the same size, based on the same whole or the same point on a number line.
3.NS.7	Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).

3.NS.8	Compare two fractions with the same numerator or the same denominator by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).
3.NS.9	Use place value understanding to round 2- and 3-digit whole numbers to the nearest 10 or 100.

COMPUTATION	
3.C.1	Fluently add and subtract whole numbers fluently within 1000 using strategies and algorithms based on place value, properties of operations, and relationships between addition and subtraction.
3.C.2	Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.
3.C.3	Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.
3.C.4	Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).
3.C.5	Multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.
3.C.6	Demonstrate fluency with mastery of multiplication facts and corresponding division facts of 0 to 10.

ALGEBRAIC THINKING	
3.AT.1	Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.2	Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.3	Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.4	Interpret a multiplication equation as equal groups (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). Represent verbal statements of equal groups as multiplication equations.
3.AT.5	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
3.AT.6	Create, extend, and give an appropriate rule for number patterns within 100 (including patterns in the addition table or multiplication table).

GEOMETRY	
3.G.1	Identify and describe the following: cube, sphere, prism, pyramid, cone, and cylinder.
3.G.2	Understand that shapes (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize and draw rhombuses, rectangles, and squares as examples of quadrilaterals. Recognize and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.3	Identify, describe and draw points, lines and line segments using appropriate tools (e.g., ruler, straightedge, and technology), and use these terms when describing two-dimensional shapes.
3.G.4	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).

MEASUREMENT	
3.M.1	Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).
3.M.2	Choose and use appropriate units and tools to estimate and measure length, weight, and temperature. Estimate and measure length to a quarter-inch, weight in pounds, and temperature in degrees Celsius and Fahrenheit.
3.M.3	Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real-world problems involving addition and subtraction of time intervals in minutes.
3.M.4	Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts using the $ symbol in the form of dollars and cents (e.g., $4.59). Solve real-world problems to determine whether there is enough money to make a purchase.
3.M.5	Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.
3.M.6	Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3.M.7	Find perimeters of polygons given the side lengths or given an unknown side length.

DATA ANALYSIS	
3.DA.1	Create scaled picture graphs, scaled bar graphs, and frequency tables to represent a data set—including data collected through observations, surveys, and experiments—with several categories. Solve one- and two-step "how many more" and "how many less" problems regarding the data and make predictions based on the data.
3.DA.2	Generate measurement data by measuring lengths with rulers to the nearest quarter of an inch. Display the data by making a line plot, where the horizontal scale is marked off in appropriate units, such as whole numbers, halves, or quarters.

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Indiana Academic Standards
Mathematics: Grade 4

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the highest-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 4

The Mathematics standards for Grade 4 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Grade 4 are made up of six strands: Number Sense, Computation, Algebraic Thinking, Geometry, Measurement, and Data Analysis. The skills listed in each strand indicate what students in Grade 4 should know and be able to do in Mathematics.

NUMBER SENSE	
4.NS.1	Read and write whole numbers up to 1,000,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000,000.
4.NS.2	Compare two whole numbers up to 1,000,000 using >, =, and < symbols.
4.NS.3	Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Name and write mixed numbers using objects or pictures. Name and write mixed numbers as improper fractions using objects or pictures.
4.NS.4	Explain why a fraction, a/b, is equivalent to a fraction, (n × a)/(n × b), by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [In grade 4, limit denominators of fractions to 2, 3, 4, 5, 6, 8, 10, 25, 100.]
4.NS.5	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark, such as 0, 1/2, and 1). Recognize comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).
4.NS.6	Write tenths and hundredths in decimal and fraction notations. Use words, models, standard form and expanded form to represent decimal numbers to hundredths. Know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 = 0.50, 7/4 = 1 3/4 = 1.75).

4.NS.7	Compare two decimals to hundredths by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).
4.NS.8	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.
4.NS.9	Use place value understanding to round multi-digit whole numbers to any given place value.

COMPUTATION	
4.C.1	Add and subtract multi-digit whole numbers fluently using a standard algorithmic approach.
4.C.2	Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Describe the strategy and explain the reasoning.
4.C.3	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning.
4.C.4	Multiply fluently within 100.
4.C.5	Add and subtract fractions with common denominators. Decompose a fraction into a sum of fractions with common denominators. Understand addition and subtraction of fractions as combining and separating parts referring to the same whole.
4.C.6	Add and subtract mixed numbers with common denominators (e.g. by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction).

4.C.7	Show how the order in which two numbers are multiplied (commutative property) and how numbers are grouped in multiplication (associative property) will not change the product. Use these properties to show that numbers can be multiplied in any order. Understand and use the distributive property.

ALGEBRAIC THINKING	
4.AT.1	Solve real-world problems involving addition and subtraction of multi-digit whole numbers (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
4.AT.2	Recognize and apply the relationships between addition and multiplication, between subtraction and division, and the inverse relationship between multiplication and division to solve real-world and other mathematical problems.
4.AT.3	Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7, and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.
4.AT.4	Solve real-world problems with whole numbers involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison. [In grade 4, division problems should not include a remainder.]
4.AT.5	Solve real-world problems involving addition and subtraction of fractions referring to the same whole and having common denominators (e.g., by using visual fraction models and equations to represent the problem).
4.AT.6	Describe a relationship between two variables and use to find a second number when a first number is given. Generate a number pattern that follows a given rule.

GEOMETRY	
4.G.1	Identify, describe, and draw parallelograms, rhombuses, and trapezoids using appropriate tools (e.g., ruler, straightedge and technology).
4.G.2	Recognize and draw lines of symmetry in two-dimensional figures. Identify figures that have lines of symmetry.
4.G.3	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.
4.G.4	Identify, describe, and draw rays, angles (right, acute, obtuse), and perpendicular and parallel lines using appropriate tools (e.g., ruler, straightedge and technology). Identify these in two-dimensional figures.
4.G.5	Classify triangles and quadrilaterals based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles (right, acute, obtuse).

MEASUREMENT	
4.M.1	Measure length to the nearest quarter-inch, eighth-inch, and millimeter.
4.M.2	Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Express measurements in a larger unit in terms of a smaller unit within a single system of measurement. Record measurement equivalents in a two-column table.
4.M.3	Use the four operations to solve real-world problems involving distances, intervals of time, volumes, masses of objects, and money. Include addition and subtraction problems involving simple fractions and problems that require expressing measurements given in a larger unit in terms of a smaller unit.
4.M.4	Apply the area and perimeter formulas for rectangles to solve real-world problems and other mathematical problems. Recognize area as additive and find the area of complex shapes composed of rectangles by decomposing them into

	non-overlapping rectangles and adding the areas of the non-overlapping parts; apply this technique to solve real-world problems and other mathematical problems.
4.M.5	Understand that an angle is measured with reference to a circle, with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. Understand an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure other angles. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.
4.M.6	Measure angles in whole-number degrees using appropriate tools. Sketch angles of specified measure.

DATA ANALYSIS	
4.DA.1	Formulate questions that can be addressed with data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, and bar graphs.
4.DA.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using data displayed in line plots.
4.DA.3	Interpret data displayed in a circle graph.

Dr. Jennifer McCormick
Superintendent of Public Instruction

DEPARTMENT OF EDUCATION

Working Together for Student Success

Indiana Academic Standards
Mathematics: Grade 5

Introduction

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PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 5

The Mathematics standards for Grade 5 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Grade 5 are made up of six strands: Number Sense, Computation, Algebraic Thinking, Geometry, Measurement, and Data Analysis and Statistics. The skills listed in each strand indicate what students in Grade 5 should know and be able to do in Mathematics.

NUMBER SENSE	
5.NS.1	Use a number line to compare and order fractions, mixed numbers, and decimals to thousandths. Write the results using >, =, and < symbols.
5.NS.2	Explain different interpretations of fractions, including: as parts of a whole, parts of a set, and division of whole numbers by whole numbers.
5.NS.3	Recognize the relationship that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right, and inversely, a digit in one place represents 1/10 of what it represents in the place to its left.
5.NS.4	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
5.NS.5	Use place value understanding to round decimal numbers up to thousandths to any given place value.
5.NS.6	Understand, interpret, and model percents as part of a hundred (e.g. by using pictures, diagrams, and other visual models).

COMPUTATION	
5.C.1	Multiply multi-digit whole numbers fluently using a standard algorithmic approach.
5.C.2	Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning used.
5.C.3	Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
5.C.4	Add and subtract fractions with unlike denominators, including mixed numbers.
5.C.5	Use visual fraction models and numbers to multiply a fraction by a fraction or a whole number.
5.C.6	Explain why multiplying a positive number by a fraction greater than one results in a product greater than the given number. Explain why multiplying a positive number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence, a/b = (n x a)/(n x b), to the effect of multiplying a/b by one.
5.C.7	Use visual fraction models and numbers to divide a unit fraction by a non-zero whole number and to divide a whole number by a unit fraction.
5.C.8	Add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value or the properties of operations. Describe the strategy and explain the reasoning.
5.C.9	Evaluate expressions with parentheses or brackets involving whole numbers using the commutative properties of addition and multiplication, associative properties of addition and multiplication, and distributive property.

ALGEBRAIC THINKING	
5.AT.1	Solve real-world problems involving multiplication and division of whole numbers (e.g. by using equations to represent the problem). In division problems that involve a remainder, explain how the remainder affects the solution to the problem.
5.AT.2	Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models and equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess whether the answer is reasonable.
5.AT.3	Solve real-world problems involving multiplication of fractions, including mixed numbers (e.g., by using visual fraction models and equations to represent the problem).
5.AT.4	Solve real-world problems involving division of unit fractions by non-zero whole numbers, and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem).
5.AT.5	Solve real-world problems involving addition, subtraction, multiplication, and division with decimals to hundredths, including problems that involve money in decimal notation (e.g. by using equations, models or drawings and strategies based on place value or properties of operations to represent the problem).
5.AT.6	Graph points with whole number coordinates on a coordinate plane. Explain how the coordinates relate the point as the distance from the origin on each axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
5.AT.7	Represent real-world problems and equations by graphing ordered pairs in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
5.AT.8	Define and use up to two variables to write linear expressions that arise from real-world problems, and evaluate them for given values.

GEOMETRY	
5.G.1	Identify, describe, and draw triangles (right, acute, obtuse) and circles using appropriate tools (e.g., ruler or straightedge, compass and technology). Understand the relationship between radius and diameter.
5.G.2	Identify and classify polygons including quadrilaterals, pentagons, hexagons, and triangles (equilateral, isosceles, scalene, right, acute and obtuse) based on angle measures and sides. Classify polygons in a hierarchy based on properties.

MEASUREMENT	
5.M.1	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step real-world problems.
5.M.2	Find the area of a rectangle with fractional side lengths by modeling with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.M.3	Develop and use formulas for the area of triangles, parallelograms and trapezoids. Solve real-world and other mathematical problems that involve perimeter and area of triangles, parallelograms and trapezoids, using appropriate units for measures.
5.M.4	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths or multiplying the height by the area of the base.
5.M.5	Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for right rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve real-world problems and other mathematical problems.
5.M.6	Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems and other mathematical problems.

DATA ANALYSIS	
5.DS.1	Formulate questions that can be addressed with data and make predictions about the data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, bar graphs, and line graphs. Recognize the differences in representing categorical and numerical data.
5.DS.2	Understand and use measures of center (mean and median) and frequency (mode), to describe a data set.

Indiana
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Indiana Academic Standards
Mathematics: Grade 6

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the highest quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards have been developed through the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 6

The Mathematics Standards for Grade 6 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Grade 6 are made up of five strands: Number Sense; Computation; Algebra and Functions; Geometry and Measurement; and Data Analysis and Statistics. The skills listed in each strand indicate what students in grade 6 should know and be able to do in Mathematics.

NUMBER SENSE	
Guiding Principle:	
6.NS.1	Understand that positive and negative numbers are used to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative numbers to represent and compare quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.2	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself (e.g., –(–3) = 3), and that 0 is its own opposite.
6.NS.3	Compare and order rational numbers and plot them on a number line. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
6.NS.4	Understand that the absolute value of a number is the distance from zero on a number line. Find the absolute value of real numbers and know that the distance between two numbers on the number line is the absolute value of their difference. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
6.NS.5	Know commonly used fractions (halves, thirds, fourths, fifths, eighths, tenths) and their decimal and percent equivalents. Convert between any two representations (fractions, decimals, percents) of positive rational numbers without the use of a calculator.

6.NS.6	Identify and explain prime and composite numbers.
6.NS.7	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers from 1 to 100, with a common factor as a multiple of a sum of two whole numbers with no common factor.
6.NS.8	Interpret, model, and use ratios to show the relative sizes of two quantities. Describe how a ratio shows the relationship between two quantities. Use the following notations: a/b, a to b, a:b.
6.NS.9	Understand the concept of a unit rate and use terms related to rate in the context of a ratio relationship.
6.NS.10	Use reasoning involving rates and ratios to model real-world and other mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

COMPUTATION	
Guiding Principle:	
6.C.1	Divide multi-digit whole numbers fluently using a standard algorithmic approach.
6.C.2	Compute with positive fractions and positive decimals fluently using a standard algorithmic approach.
6.C.3	Solve real-world problems with positive fractions and decimals by using one or two operations.

6.C.4	Compute quotients of positive fractions and solve real-world problems involving division of fractions by fractions. Use a visual fraction model and/or equation to represent these calculations.
6.C.5	Evaluate positive rational numbers with whole number exponents.
6.C.6	Apply the order of operations and properties of operations (identity, inverse, commutative properties of addition and multiplication, associative properties of addition and multiplication, and distributive property) to evaluate numerical expressions with nonnegative rational numbers, including those using grouping symbols, such as parentheses, and involving whole number exponents.

ALGEBRA AND FUNCTIONS	
Guiding Principle:	
6.AF.1	Evaluate expressions for specific values of their variables, including expressions with whole-number exponents and those that arise from formulas used in geometry and other real-world problems.
6.AF.2	Apply the properties of operations (e.g., identity, inverse, commutative, associative, distributive properties) to create equivalent linear expressions and to justify whether two linear expressions are equivalent when the two expressions name the same number regardless of which value is substituted into them.
6.AF.3	Define and use multiple variables when writing expressions to represent real-world and other mathematical problems, and evaluate them for given values.
6.AF.4	Understand that solving an equation or inequality is the process of answering the following question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.AF.5	Solve equations of the form x + p = q, x - p = q, px = q, and x/p = q fluently for cases in which p, q and x are all nonnegative rational numbers. Represent real-world problems using equations of these forms and solve such problems.
6.AF.6	Write an inequality of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$, where c is a rational number, to represent a constraint or condition in a real-world or other mathematical problem. Recognize inequalities have infinitely many solutions and represent solutions on a number line diagram.
6.AF.7	Understand that signs of numbers in ordered pairs indicate the quadrant containing the point. Identify rules or patterns in the signs as they relate to the quadrantsGraph points with rational number coordinates on a coordinate plane.
6.AF.8	Solve real-world and other mathematical problems by graphing points with rational number coordinates on a coordinate plane. Include the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
6.AF.9	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane.
6.AF.10	Use variables to represent two quantities in a proportional relationship in a real-world problem; write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

GEOMETRY AND MEASUREMENT	
Guiding Principle:	
6.GM.1	Convert between measurement systems (English to metric and metric to English) given conversion factors, and use these conversions in solving real-world problems.

6.GM.2	Know that the sum of the interior angles of any triangle is 180º and that the sum of the interior angles of any quadrilateral is 360º. Use this information to solve real-world and mathematical problems.
6.GM.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate; apply these techniques to solve real-world and other mathematical problems.
6.GM.4	Find the area of complex shapes composed of polygons by composing or decomposing into simple shapes; apply this technique to solve real-world and other mathematical problems.
6.GM.5	Find the volume of a right rectangular prism with fractional edge lengths using unit cubes of the appropriate unit fraction edge lengths (e.g., using technology or concrete materials), and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = Bh to find volumes of right rectangular prisms with fractional edge lengths to solve real-world and other mathematical problems.
6.GM.6	Construct right rectangular prisms from nets and use the nets to compute the surface area of prisms; apply this technique to solve real-world and other mathematical problems.

DATA ANALYSIS AND STATISTICS	
Guiding Principle:	
6.DS.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for the variability in the answers. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.DS.2	Select, create, and interpret graphical representations of numerical data, including line plots, histograms, and box plots.
6.DS.3	Formulate statistical questions; collect and organize the data (e.g., using technology); display and interpret the data with graphical representations (e.g., using technology).

6.DS.4	Summarize numerical data sets in relation to their context in multiple ways, such as: ● report the number of observations ● describe the nature of the attribute under investigation, including how it was measured and its units of measurement ● determine quantitative measures of center (mean and/or median) and spread (range and interquartile range) ● describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered ● relate the choice of measures of center and spread to the shape of the data distribution and the context in which the data were gathered

DEPARTMENT OF EDUCATION

Working Together for Student Success

Indiana Academic Standards
Mathematics: Grade 7

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the highest quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

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PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 7

The Mathematics Standards for Grade 7 are supplemented by the Process Standards for Mathematics.

The Mathematics Standards for Grade 7 are made up of five strands: Number Sense; Computation; Algebra and Functions; Geometry and Measurement; and Data Analysis, Statistics, and Probability. The skills listed in each strand indicate what students in grade 7 should know and be able to do in Mathematics.

NUMBER SENSE	
Guiding Principle:	
7.NS.1	Find the prime factorization of whole numbers and write the results using exponents.
7.NS.2	Understand the inverse relationship between squaring and finding the square root of a perfect square whole number. Find square roots of perfect square whole numbers.
7.NS.3	Know there are rational and irrational numbers. Identify, compare, and order rational and irrational numbers (e.g. √2, √3, √5, Π) and plot them on a number line.

COMPUTATION	
Guiding Principle:	
7.NS.1	Understand p + q as the number located a distance |q| from p, in the positive or negative direction, depending on whether q is positive or negative. Show on a number line that a number and its opposite have a sum of 0 (are additive inverses). Find and interpret sums of rational numbers in real-world contexts.

7.NS.2	Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
7.NS.3	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers.
7.NS.4	Understand that integers can be divided, provided that the divisor is not zero. Understand that if p and q are integers, then –(p/q) = (–p)/q = p/(–q).
7.NS.5	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
7.NS.6	Use proportional relationships to solve ratio and percent problems with multiple operations (e.g. simple interest, tax, markups, markdowns, gratuities, conversions within and across measurement systems, and percent increase and decrease).
7.NS.7	Compute fluently with rational numbers using an algorithmic approach.
7.NS.8	Solve real-world problems with rational numbers by using one or two operations.

ALGEBRA AND FUNCTIONS	
Guiding Principle:	
7.AF.1	Apply the properties of operations (e.g., identity, inverse, commutative, associative, distributive properties) to create equivalent linear expressions, including situations that involve factoring out a common number (e.g., given 2x - 10, create an equivalent expression 2(x - 5)). Justify each step in the process.

7.AF.2	Solve equations of the form px + q= r and p(x + q) = r fluently, where p, q, and r are specific rational numbers. Represent real-world problems using equations of these forms and solve such problems.
7.AF.3	Solve inequalities of the form px +q (> or ≥) r or px + q (< or ≤) r, where p, q, and r are specific rational numbers. Represent real-world problems using inequalities of these forms and solve such problems. Graph the solution set of the inequality and interpret it in the context of the problem.
7.AF.4	Define slope as vertical change for each unit of horizontal change and recognize that a constant rate of change or constant slope describes a linear function. Identify and describe situations with constant or varying rates of change.
7.AF.5	Graph a line given its slope and a point on the line. Find the slope of a line given its graph.
7.AF.6	Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).
7.AF.7	Identify the unit rate or constant of proportionality in tables, graphs, equations, and verbal descriptions of proportional relationships.
7.AF.8	Explain what the coordinates of a point on the graph of a proportional relationship mean in terms of the situation, with special attention to the points (0, 0) and (1,r), where r is the unit rate.
7.AF.9	Represent real-world and other mathematical situations that involve proportional relationships. Write equations and draw graphs to represent these proportional relationships. Recognize that these situations are described by a linear function in the form y = mx, where the unit rate, m, is the slope of the line.

GEOMETRY AND MEASUREMENT
Guiding Principle:

7.GM.1	Explore triangles with given conditions from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.GM.2	Identify and describe similarity relationships of polygons including the angle-angle criterion for similar triangles, and solve problems involving similarity.
7.GM.3	Solve real-world and other mathematical problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing. Create a scale drawing by using proportional reasoning.
7.GM.4	Solve real-world and other mathematical problems using facts about vertical, adjacent, complementary, and supplementary angles.
7.GM.5	Understand the formulas for area and circumference of a circle and use them to solve real-world and other mathematical problems; give an informal derivation of the relationship between circumference and area of a circle.
7.GM.6	Solve real-world and other mathematical problems involving volume of cylinders and three-dimensional objects composed of right rectangular prisms.
7.GM.7	Construct nets for right rectangular prisms and cylinders and use the nets to compute the surface area; apply this technique to solve real-world and other mathematical problems.

DATA ANALYSIS, STATISTICS, AND PROBABILITY	
Guiding Principle:	
7.DSP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population. Understand that conclusions and generalizations about a population from a sample are valid only if the sample is representative of that population and that random sampling tends to produce representative samples and support valid inferences.

7.DSP.2	Use data from a random sample to draw inferences about a population. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
7.DSP.3	Find, use, and interpret measures of center (mean and median) and measures of spread (range, interquartile range, and mean absolute deviation) for numerical data from random samples to draw comparative inferences about two populations.
7.DSP.4	Make observations about the degree of visual overlap of two numerical data distributions represented in line plots or box plots. Describe how data, particularly outliers, added to a data set may affect the mean and/or median.
7.DSP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Understand that a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. Understand that a probability of 1 indicates an event certain to occur and a probability of 0 indicates an event impossible to occur. Identify probabilities of events as impossible, unlikely, equally likely, likely, or certain.
7.DSP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its relative frequency from a large sample.
7.DSP.7	Develop probability models that include the sample space and probabilities of outcomes to represent simple events with equally likely outcomes. Predict the approximate relative frequency of the event based on the model. Compare probabilities from the model to observed frequencies; evaluate the level of agreement and explain possible sources of discrepancy.

Indiana
DEPARTMENT OF EDUCATION
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Indiana Academic Standards
Mathematics: Grade 8

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the highest quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

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PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Grade 8

The Mathematics Standards for Grade 8 are supplemented by the Process Standards for Mathematics.

The Mathematics Standards for Grade 8 are made up of five strands: Number Sense; Computation; Algebra and Functions; Geometry and Measurement; and Data Analysis, Statistics, and Probability. The skills listed in each strand indicate what students in grade 8 should know and be able to do in mathematics.

NUMBER SENSE	
Guiding Principle:	
8.NS.1	Give examples of rational and irrational numbers and explain the difference between them. Understand that every number has a decimal equivalent. For rational numbers, show that the decimal equivalent terminates or repeats, and convert a repeating decimal into a rational number.
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, plot them approximately on a number line, and estimate the value of expressions involving irrational numbers.
8.NS.3	Given a numeric expression with common rational number bases and integer exponents, apply the properties of exponents to generate equivalent expressions.
8.NS.4	Use square root symbols to represent solutions to equations of the form x^2 = p, where p is a positive rational number.

COMPUTATION
Guiding Principle:

8.C.1	Solve real-world problems with rational numbers by using multiple operations.
8.C.2	Solve real-world and other mathematical problems involving numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Interpret scientific notation that has been generated by technology, such as a scientific calculator, graphing calculator, or excel spreadsheet.

GEOMETRY AND MEASUREMENT	
Guiding Principle:	
8.AF.1	Solve linear equations and inequalities with rational number coefficients fluently, including those whose solutions require expanding expressions using the distributive property and collecting like terms. Represent real-world problems using linear equations and inequalities in one variable and solve such problems.
8.AF.2	Generate linear equations in one variable with one solution, infinitely many solutions, or no solutions. Justify the classification given.
8.AF.3	Understand that a function assigns to each x-value (independent variable) exactly one y-value (dependent variable), and that the graph of a function is the set of ordered pairs (x,y).
8.AF.4	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, has a maximum or minimum value). Sketch a graph that exhibits the qualitative features of a function that has been verbally described.
8.AF.5	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. Describe similarities and differences between linear and nonlinear functions from tables, graphs, verbal descriptions, and equation
8.AF.6	Construct a function to model a linear relationship between two quantities given a verbal description, table of values, or graph. Recognize in y = mx + b that m is the slope (rate of change) and b is the y-intercept of the graph, and describe the meaning of each in the context of a problem.

8.AF.7	Compare properties of two linear functions given in different forms, such as a table of values, equation, verbal description, and graph (e.g., compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed).
8.AF.8	

GEOMETRY AND MEASUREMENT	
Guiding Principle:	
8.GM.1	Identify, define, and describe attributes of three-dimensional geometric objects (right rectangular prisms, cylinders, cones, spheres, and pyramids). Explore the effects of slicing these objects using appropriate technology and describe the two-dimensional figure that results.
8.GM.2	Solve real-world and other mathematical problems involving volume of cones, spheres, and pyramids and surface area of spheres.
8.GM.3	Verify experimentally the properties of rotations, reflections, and translations, including: lines are mapped to lines, and line segments to line segments of the same length; angles are mapped to angles of the same measure; and parallel lines are mapped to parallel lines.
8.GM.4	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Describe a sequence that exhibits the congruence between two given congruent figures.
8.GM.5	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Describe a sequence that exhibits the similarity between two given similar figures.
8.GM.6	Explore dilations, translations, rotations, and reflections on two-dimensional figures in the coordinate plane.

8.GM.7	Use inductive reasoning to explain the Pythagorean relationship.
8.GM.8	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and other mathematical problems in two dimensions.
8.GM.9	Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane.

DATA ANALYSIS, STATISTICS, AND PROBABILITY	
Guiding Principle:	
8.DSP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantitative variables. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
8.DSP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and describe the model fit by judging the closeness of the data points to the line.
8.DSP.3	Write and use equations that model linear relationships to make predictions, including interpolation and extrapolation, in real-world situations involving bivariate measurement data. ~~; I~~Interpret the slope and y-intercept in context.
8.DSP.4	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Understand and use appropriate terminology to describe independent, dependent, complementary, and mutually exclusive events.
8.DSP.5	Represent sample spaces and find probabilities of compound events (independent and dependent) using ~~methods, such as~~ organized lists, tables, and tree diagrams.

8.DSP.6	For events with a large number of outcomes, understand the use of the multiplication counting principle. Develop the multiplication counting principle and apply it to situations with a large number of outcomes.

Indiana
DEPARTMENT OF EDUCATION
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Indiana Academic Standards
Mathematics: Kindergarten

Introduction

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What are the Indiana Academic Standards?

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MATHEMATICS: Kindergarten

The Mathematics standards for Kindergarten are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Kindergarten are made up of five strands: Number Sense, Computation and Algebraic Thinking, Geometry, Measurement, and Data Analysis. The skills listed in each strand indicate what students in Kindergarten should know and be able to do in Mathematics.

NUMBER SENSE	
K.NS.1	Count to at least 100 by ones and tens and count on by one from any number.
K.NS.2	Write whole numbers from zero to 20 and recognize number words from zero to 10. Represent a number of objects with a written numeral zero to 20 (with zero representing a count of no objects).
K.NS.3	Find the number that is one more than or one less than any whole number up to 20.
K.NS.4	Say the number names in standard order when counting objects, pairing each object with one and only one number name and each number name with one and only one object. Understand that the last number name said describes the number of objects counted and that the number of objects is the same regardless of their arrangement or the order in which they were counted.
K.NS.5	Count up to 20 objects arranged in a line, a rectangular array, or a circle. Count up to 10 objects in a scattered configuration. Count out the number of objects, given a number from one to 20.
K.NS.6	Recognize sets of one to 10 objects in patterned arrangements and tell how many without counting.
K.NS.7	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g. by using matching and counting strategies).

K.NS.8	Compare the values of two numbers from 1 to 20 presented as written numerals.
K.NS.9	Correctly use the words for comparison, including: one and many; none, some and all; more and less; most and least; and equal to, more than and less than.
K.NS.10	Separate sets of 10 or fewer objects into equal groups.
K.NS.11	Develop initial understandings of place value and the base 10 number system by showing equivalent forms of whole numbers from 10 to 20 as groups of tens and ones using objects and drawings.

COMPUTATION AND ALGEBRAIC THINKING	
K.CA.1	Use objects, drawings, mental images, sounds, etc., to represent addition and subtraction within 10.
K.CA.2	Solve real-world problems that involve addition and subtraction within 10 (e.g., by using objects or drawings to represent the problem).
K.CA.3	Use objects, drawings, etc., to decompose numbers less than or equal to 10 into pairs in more than one way, and record each decomposition with a drawing or an equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). [In Kindergarten, students should see equations and be encouraged to trace them, however, writing equations is not required.]
K.CA.4	Find the number that makes 10 when added to the given number for any number from one to nine (e.g., by using objects or drawings), and record the answer with a drawing or an equation.
K.CA.5	Create, extend, and give an appropriate rule for simple repeating and growing patterns with numbers and shapes.

GEOMETRY	
K.G.1	Describe the positions of objects and geometric shapes in space using the terms inside, outside, between, above, below, near, far, under, over, up, down, behind, in front of, next to, to the left of and to the right of.
K.G.2	Compare two- and three-dimensional shapes in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
K.G.3	Model shapes in the world by composing shapes from objects (e.g., sticks and clay balls) and drawing shapes.
K.G.4	Compose simple geometric shapes to form larger shapes (e.g., create a rectangle composed of two triangles).

MEASUREMENT	
K.M.1	Make direct comparisons of the length, capacity, weight, and temperature of objects, and recognize which object is shorter, longer, taller, lighter, heavier, warmer, cooler, or holds more.
K.M.2	Understand concepts of time, including: morning, afternoon, evening, today, yesterday, tomorrow, day, week, month, and year. Understand that clocks and calendars are tools that measure time.

DATA ANALYSIS	
K.DA.1	Identify, sort, and classify objects by size, number, and other attributes. Identify objects that do not belong to a particular group and explain the reasoning used.

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Indiana Academic Standards
Mathematics: Precalculus: Algebra

Introduction

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	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
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PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Precalculus: Algebra

Functions	
Guiding Principle:	
PC.F.1	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
PC.F.2	Find linear models by using median fit and least squares regression methods, making use of technology. Decide which among several linear models gives a better fit. Interpret the slope and intercept in terms of the original context.
PC.F.3	Compose functions and find the domain of composite functions.
PC.F.4	Determine if a graph or table has an inverse, and justify if the inverse is a function, relation, or neither. Identify the values of an inverse function/relation from a graph or a table, given that the function has an inverse. Derive the inverse equation from the values of the inverse.
PC.F.5	Produce an invertible function from a non-invertible function by restricting the domain.
PC.F.6	Recognize even and odd functions from their graphs and algebraic expressions.

Quadratic, Polynomial and Rational Equations and Functions	
Guiding Principle:	
PC.QPR.1	Use the method of completing the square to transform any quadratic equation into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
PC.QPR.2	Understand and use addition, subtraction, multiplication, and conjugation of complex numbers.
PC.QPR.3	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
PC.QPR.4	Know and apply the Remainder Theorem and the Factor Theorem.
PC.QPR.5	Understand the Fundamental Theorem of Algebra. Find a polynomial function of lowest degree with real coefficients when given its roots.
PC.QPR.6	Graph rational functions with and without technology. Identify and describe features such as intercepts, domain and range, and asymptotic and end behavior.

Exponential and Logarithmic Functions	
Guiding Principle:	
PC.EL.1	Use the definition of logarithms to convert logarithms from one base to another and prove simple laws of logarithms.
PC.EL.2	Use the laws of logarithms to simplify logarithmic expressions, approximate the value of a logarithmic expression, and solve logarithmic equations.

PC.EL.3	Graph and solve real-world and other mathematical problems that can be modeled using exponential and logarithmic functions; interpret the solution and determine whether it is reasonable. Identify and describe features such as intercepts, domain, range, asymptotes, and end behavior.
PC.EL.4	Use technology to find a quadratic, exponential, logarithmic, or power function that models a relationship for a bivariate data set to make predictions.

Sequences and Series	
Guiding Principle:	
PC.SS.1	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
PC.SS.2	Write arithmetic and geometric sequences both recursively and with an explicit formula; use them to model situations and translate between the two forms.
PC.SS.3	Find partial sums of arithmetic and geometric series and represent them using sigma notation.
PC.SS.4	Model and solve real-world problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable.

Conics	
Guiding Principle:	
PC.CO.1	Construct the equation of a parabola given a focus and directrix.
PC.CO.2	Construct the equation of a circle of given center and radius. Complete the square to find the center and radius of a circle given by an equation.
PC.CO.3	Construct the equations of ellipses and hyperbolas given at least 2 of the following: foci, vertices, length of an axis, or point on the curve.
PC.CO.4	Graph conic sections. Identify and describe features like center, vertex or vertices, focus or foci, directrix, axis of symmetry, major axis, minor axis, and eccentricity.

Indiana
DEPARTMENT OF EDUCATION
Working Together for Student Success

Indiana Academic Standards
Mathematics: PreCalculus: Trigonometry

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases

	and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.

PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern of structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
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MATHEMATICS: Precalculus: Trigonometry

Unit Circle	
Guiding Principle:	
TR.UC.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
TR.UC.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
TR.UC.3	Use special triangles to determine the values of sine, cosine, and tangent for π/3, π/4, and π/6. Apply special right triangles to the unit circle and use them to express the values of sine, cosine, and tangent for x, π ± x, and 2π ± x in terms of their values for x, where x is any real number.

Triangles	
Guiding Principle:	
TR.T.1	Define and use the trigonometric ratios (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles and the coordinates on the unit circle.
TR.T.2	Solve real-world problems with and without technology that can be modeled using right triangles, including problems that can be modeled using trigonometric ratios. Interpret the solutions and determine whether the solutions are reasonable.
TR.T.3	Explain and use the relationship between the sine and cosine of complementary angles

TR.T.4	Prove the Laws of Sines and Cosines.
TR.T.5	Understand and apply the Laws of Sines and Cosines to solve real-world and other mathematical problems involving right and non-right triangles.
TR.T.6	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line. Use the formula to find areas of triangles.

Periodic Functions	
Guiding Principle:	
TR.PF.1	Graph trigonometric functions with and without technology. Use the graphs to model and analyze periodic phenomena, stating amplitude, period, frequency, phase shift, and midline (vertical shift).
TR.PF.2	Model a data set with periodicity using a sinusoidal function and explain the parameters of the model.
TR.PF.3	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
TR.PF.4	Construct the inverse trigonometric functions of sine, cosine, and tangent by restricting the domain.
TR.PF.5	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Identities	
Guiding Principle:	
TR.ID.1	Prove the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ and use it to find trigonometric ratios, given sin(x), cos(x), or tan(x), and the quadrant of the angle.
TR.ID.2	Verify trigonometric identities and simplify expressions using trigonometric identities.
TR.ID.3	Prove the addition and subtraction identities for sine, cosine, and tangent. Use the identities to solve problems.
TR.ID.4	Prove the double- and half-angle identities for sine, cosine, and tangent. Use the identities to solve problems.

Polar Coordinates and Complex Numbers	
Guiding Principle:	
TR.PC.1	Understand and use complex numbers, including real and imaginary numbers, on the complex plane in rectangular and polar form, and explain why the rectangular and polar forms of a given complex number represent the same number.
TR.PC.2	State, prove, and use DeMoivre's Theorem.
TR.PC.3	Define polar coordinates and relate polar coordinates to Cartesian coordinates.
TR.PC.4	Translate equations from rectangular coordinates to polar coordinates and from polar coordinates to rectangular coordinates. Graph equations in the polar coordinate plane.

Vectors	
Guiding Principle:	
TR.V.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$).
TR.V.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
TR.V.3	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
TR.V.4	Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
TR.V.5	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v}x, \mathbf{v}y) = (c\mathbf{v}x, c\mathbf{v}y)$.
TR.V.6	Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\lVert c\mathbf{v}\rVert = \lvert c\rvert\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $\lvert c\rvert\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).
TR.V.7	Solve problems involving velocity and other quantities that can be represented by vectors.