MEMORANDUM

To: Indiana State Board of Education

From: Office of School Improvement, Indiana Department of Education

Date: December 3, 2018

Re: Analytical Algebra II Approval

Pursuant to IC 20-30-10-2.5 (a) and (b) the Indiana State Board of Education shall consider math course requirements other than Algebra II for the Core 40 designation. Any such course adopted "must be at a level of difficulty that aligns with postsecondary preparation." Additionally, if a school offers such a course and if a student plans to enroll in this course, the parent and student must sign a consent form, developed by the state board in collaboration with the commission for higher education, notifying the parent and the student that enrollment in this course may affect the student's ability to attend a particular postsecondary educational institution or enroll in a particular course at a particular postsecondary educational institution because the course may not align with academic requirements established by the postsecondary educational institution.

A committee of stakeholders was convened to consider alternatives to Algebra II for the Core 40 diploma designation. Following the parameters given in IC 20-30-10-2.5 (a), standards for Analytical Algebra II were developed. MetaMetrics provided a report establishing the level of difficulty and the Dana Center provided feedback on the content. Additionally, public feedback was sought and incorporated into the final course proposal. The course description for Analytical Algebra II as provided in the Course Titles and Descriptions is as follows:

ANALYTICAL ALGEBRA II
2524 (ANA ALG)

Analytical Algebra II builds on previous work with linear, quadratic and exponential functions and extends to include polynomial, rational, radical, logarithmic, and other functions. Data analysis, statistics, and probability content should be included throughout the course, as students collect and use univariate and bivariate data to create and interpret mathematical models. Additionally, Analytical Algebra II should focus on the application of mathematics in various disciplines including business, finance, science, career and technical education, and social sciences, using technology to model real-world problems with various functions, using and translating between multiple representations. The eight Process Standards for Mathematics apply throughout the course. Together with the content standards, the Process Standards prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. This course is not recommended for students interested in pursuing a STEM degree at a four year institution; this course may not prepare students for PreCalculus/Trigonometry.

- Recommended Grade Level: 9, 10, 11, 12
- Recommended Prerequisite: Algebra I
- Credits: 2 semester course, 1 credit per semester
- Fulfills the Algebra II/Integrated Mathematics III requirement for all diplomas; if students use this course to fulfill this credit, the parent and student must sign a consent form notifying the parent and the student that enrollment in Analytical Algebra II may affect the student's ability to attend a particular postsecondary educational institution or enroll in a particular course at a particular postsecondary educational institution because Analytical Algebra II may not align with academic requirements established by the postsecondary educational institution.

Additionally, the successful completion of Analytical Algebra II would allow a student to enroll in Finite Mathematics, Probability and Statistics, or Quantitative Reasoning.

Having met the parameters established by Indiana Code, the Department recommends the Board approve Analytical Algebra II as an alternative to the Algebra II requirement in the Core 40 curriculum.

ANALYTICAL ALGEBRA II

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

Analytical Algebra II should focus on the application of mathematics in various disciplines including business, finance, science, career and technical education, and social sciences. This course covers most of the traditional Algebra II standards, but the focus is on the application of algebraic concepts rather than theoretical concepts.

Building on previous work with linear, quadratic, and exponential functions, Analytical Algebra II should extend to include polynomial, rational, radical, logarithmic, and other functions. Students should be able to model real-world problems with various functions using and translating between multiple representations. Additionally, students should be able to interpret key features of function models within a given context. Students should also build on previous work done with data analysis, statistics, including univariate and bivariate data, and probability.

ANALYTICAL ALGEBRA II

Guiding Principle: Data analysis, statistics, and probability content should be included throughout the course, as students collect and use univariate and bivariate data to create and interpret mathematical models. They should be able to make inferences and justify conclusions from various experimental and survey data, and develop a basic understanding of the structure of a good study, the biases that might exist, and the importance of randomization.

Data Analysis, Statistics, and Probability	AA.DSP.1: Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization and possible sources of bias relate to each.
	AA.DSP.2: Choose, create, and critique, with technology, mathematical models (linear, quadratic and exponential) for bivariate data sets. Use the models to interpolate and/or extrapolate, to answer questions, and to draw conclusions or make decisions, addressing limitations and long-term ramifications. Recognize when a change in model is needed. Interpret the correlation coefficient for linear models.
	AA.DSP.3: Read, interpret, and make decisions about data summarized numerically using measures of center and spread, in tables, and in graphical displays (line graphs, bar graphs, scatterplots, and histograms), e.g., explain why the mean may not represent a typical salary; critique a graphical display by recognizing that the choice of scale can distort information.
	AA.DSP.4: Analyze and compare univariate data of two or more different data sets using measures of center (mean, median, and mode), shape, and spread (range, interquartile range, standard deviation, percentiles, and variance) making use of technology. Understand the effects of outliers on the statistical summary of the data.
	AA.DSP.5: Record multiple observations (or simulated samples) of random events and construct empirical models of the probability distributions. Construct a theoretical model and apply the law of large numbers to show the relationship between the two models.

	AA.DSP.6: Evaluate the validity of claims based on empirical probabilities and theoretical probabilities, including those derived from dependent and independent events. Draw conclusions and make decisions in various probabilistic contexts. Make use of different representations of data including two-way tables and tree diagrams.
	AA.DSP.7: Determine the nature and number of elements in a finite sample space to model the outcomes of real-world events using the multiplication counting principle, permutations, and combinations.

Guiding Principle: Extending from work with linear functions in Algebra I, this content should include work with arithmetic sequences and series, understanding the relationship to linear functions. Additionally, students should solidify their understanding of systems of equations. The focus should be on solving systems of equations that represent real-world situations, with technology. Students should be able to solve systems that involve non-linear equations. They should also be able to solve systems of equations with three variables with technology, using various strategies such as matrices.

Linear Functions and Beyond	AA.LF.1: Model real world situations involving arithmetic sequences and understand that they can be defined both recursively and with an explicit formula.
	AA.LF.2: Find partial sums of arithmetic series that model real world situations. Understand and apply partial sums of arithmetic series written in sigma notation.
	AA.LF.3: Recognize functional relationships in real world contexts. Translate fluently among multiple representations (graphs, tables, equations, and verbal descriptions).
	AA.LF.4: Within real world contexts, understand composition of functions and combine functions by composition.
	AA.LF.5: Describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative) with technology. Find the value of k given the graph of f(x) and the graph of f(x) + k, k f(x), f(kx), or f(x + k). Identify and analyze transformations within a real world context.

	AA.LF.6: Solve a system of equations and/or inequalities consisting of a linear equation and a quadratic equation in two variables algebraically and graphically with technology (e.g., find the points of intersection between the line y = –3x and the parabola $y=x^2 - 6x-5$).
	AA.LF.7: Represent real-world problems using a system of linear equations and/or inequalities in two or three variables. Solve such systems graphically or with matrices, as appropriate to the system, with technology. Interpret the solution and determine whether it is reasonable.
Guiding Principle: Extending from Algebra I, students should be able to represent real-world problems that can be modeled with quadratic or higher-order polynomial functions, interpreting key attributes in a given context.	
Quadratic and Other Polynomial Functions	AA.QP.1: Represent real-world problems that can be modeled with quadratic functions using tables, graphs, and equations; translate fluently among these representations. Solve such problems with technology. Interpret the solutions and determine whether they are reasonable.
	AA.QP.2: Rewrite quadratic functions into the form $y = a(x - h)^2 + k$ using a variety of strategies and graph these functions with technology. Understand that different forms of an equation can provide different information. Identify and interpret within a given context intercepts, zeros, domain and range, and lines of symmetry.
	AA.QP.3: Use the discriminant to determine the number and type of solutions of a quadratic equation in one variable with real coefficients. Know there is an imaginary number, i, such that $i^2 = -1$, and understand the relationship to non-real complex roots.
	AA.QP.4: Represent real-world problems that can be modeled with polynomial functions using graphs and equations. Solve such problems with technology. Interpret the solutions and determine whether they are reasonable.

	AA.QP.5: Graph polynomial functions that model a real-world situation with technology. Identify, describe, and interpret key features in the context of the situation, such as intercepts, zeros, domain and range, end behavior, maxima and minima, and lines of symmetry.
Guiding Principle: Extending from initial work with exponential functions in Algebra I, students should understand the relationship between logarithmic and exponential functions. Additionally, this content should include representing real-world problems that can be modeled with either exponential or logarithmic functions, interpreting key attributes in a given context. Arithmetic and geometric sequences and series should also be introduced, making the connection to linear and exponential functions respectively.	
Exponential and Logarithmic Functions	AA.EL.1: Model real world situations involving geometric sequences and understand that they can be defined both recursively and with an explicit formula.
	AA.EL.2: Find partial sums of geometric series that model real world situations. Understand and apply partial sums of geometric series written in sigma notation.
	AA.EL.3: Represent real-world problems using exponential functions in one or two variables and solve such problems with and without technology. Interpret the solutions and determine whether they are reasonable.
	AA.EL.4: Graph exponential functions that model real-world situations with technology. Identify, describe, and interpret key features, such as intercepts, zeros, domain, range, asymptotic and end behavior.
	AA.EL.5: Given real-world contexts, identify the percent rate of change in exponential functions written as equations, such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{(12t)}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Analyze growth and decay using absolute and relative change and make comparisons using absolute and relative difference.
	AA.EL.6: Use the properties of exponents to transform expressions for exponential functions in a given real-world context. (e.g., the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%).

	AA.EL.7: Know that the inverse of an exponential function is a logarithmic function. Represent exponential and logarithmic functions that model real-world situations using graphing technology and describe their inverse relationship. Use the inverse relationship between exponential functions and logarithms to evaluate expressions and solve equations in one variable.
Guiding Principle: This content should include representing real-world problems that can be modeled with rational, radical, and piecewise-defined functions. Students should be able to translate between various representations and interpret key attributes in a given context.	
Rational, Radical, and Other Functions	AA.R.1: Represent and solve real-world problems that can be modeled with rational functions (including direct, inverse, and joint variation) using tables, graphs, and equations; translate among these representations. Graph rational functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior.
	AA.R.2: Represent and solve real-world problems that can be modeled with radical functions using tables, graphs, and equations; translate among these representations. Graph radical functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior.
	AA.R.3: Represent and solve real-world problems that can be modeled with piecewise-defined functions (including step functions and absolute value functions) using tables, graphs, and equations; translate among these representations. Graph piecewise-defined functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior.
	AA.R.4: Translate expressions between radical and exponent form and simplify them using the laws of exponents. Understand that, while they name the same expression, one form may be more advantageous than another given the context.

Crosswalk between Algebra II and Analytical Algebra II

This crosswalk starts with the Algebra II standards and shows how they correspond to the Analytical Algebra II Standards. However, it is important to note that Guiding Principles were added to the Analytical Algebra II course to provide additional context. Additionally, the order of the standards was rearranged. While the order is not dictated by the standards, often a subtle message is sent regarding what is most important. Thus, we attempted to address this with the creation and arrangement of the Analytical Algebra standards.

More specifically, with Analytical Algebra, we moved the Data Analysis, Statistics, and Probability standards to the beginning of the document. We wanted to send the message that the other standards should be considered through the lens of data and contexts, and that these standards should be woven throughout the course. We then arranged the standards within function families, beginning with Linear Functions and Beyond, Quadratic and Other Polynomial Functions, Exponential and Logarithmic Functions, and Rational, Radical and Other Functions. We want students to understand the world around them, to analyze data, create models, and make and justify conclusions.

ALGEBRA II		Analytical Algebra II	Changes
COMPLEX NUMBERS AND EXPRESSIONS	AII.CNE.1: Know there is an imaginary number, i, such that $i^2 = -1$, and every complex number can be written in the form a + bi, with a and b real. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	AA.QP.3: Use the discriminant to determine the number and type of solutions of a quadratic equation in one variable with real coefficients. Know there is an imaginary number, i, such that $i^2 = -1$, and understand the relationship to non-real complex roots.	This standard was blended with AII.Q.3 in AA.QP.3 with the focus on knowing there is an imaginary number and understanding its relationship to non-real complex roots. Students in Analytical Algebra II are no longer going to be asked to compute with complex numbers.

	AII.CNE.2: Translate expressions between radical and exponent form and simplify them using the laws of exponents.	AA.R.4: Translate expressions between radical and exponent form and simplify them using the laws of exponents. Understand that, while they name the same expression, one form may be more advantageous than another given the context.	Emphasis in regards to understanding the equivalence of both forms and the benefits and advantages of each form dependent upon the context.
	AII.CNE.3: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide algebraic rational expressions.		The Algebra II standard is very theoretical and limited in application, tending to symbol manipulation. It was omitted from Analytical Algebra II.
	AII.CNE.4: Rewrite algebraic rational expressions in equivalent forms (e.g., using laws of exponents and factoring techniques).		The Algebra II standard is very theoretical and limited in application, tending to symbol manipulation. It was omitted from Analytical Algebra II.

	AII.CNE.5: Rewrite rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using long division and synthetic division.		The Algebra II standard is very theoretical and limited in application, tending to symbol manipulation. It was omitted from Analytical Algebra II.
	AII.CNE.6: Find partial sums of arithmetic and geometric series and represent them using sigma notation.	AA.LF.2: Find partial sums of arithmetic series that model real world situations. Understand and apply partial sums of arithmetic series written in sigma notation. AA.EL.2: Find partial sums of geometric series that model real world situations. Understand and apply partial sums of geometric series written in sigma notation.	Emphasis on real world situations and connections to linear functions (for arithmetic sequences) and exponential functions (for geometric sequences). Introduction to sigma notation, but it should be limited to basic understanding and application.
FUNCTIONS	AII.F.1: Determine whether a relation represented by a table, graph, or equation is a function.	AA.LF.3: Recognize functional relationships in real world contexts. Translate fluently among multiple representations (graphs, tables, equations, and verbal descriptions).	The Algebra II standard is written very theoretically. Students know what a function is in Algebra I, so it is somewhat repetitive. In Analytical Algebra II the focus is on recognizing functional relationships in real world contexts. Students should also be able to translate between multiple representations.

	AII.F.2: Understand composition of functions and combine functions by composition.	AA.LF.4: Within real world contexts, understand composition of functions and combine functions by composition.	The Algebra II standard is very theoretical and limited in application, tending to symbol manipulation. In Analytical Algebra II the emphasis is on composition of functions in real world contexts, such as composing the profit function of a company from the demand function of the product.
	AII.F.3: Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as f and g are inverse functions if and only if f(x)=y and g(y)=x, for all values of x in the domain of f and all values of y in the domain of g. Find the inverse of a function that has an inverse.		The Algebra II standard is very theoretical and limited in application, tending to symbol manipulation. It was omitted from Analytical Algebra II. However, students in Analytical Algebra II will be exposed to specific inverse functions such as exponential and logarithmic functions.
	AII.F.4: Understand that if the graph of a function contains a point (a, b), then the graph of the inverse relation of the function contains the point (b, a); the inverse is a reflection over the line y = x.		The Algebra II standard is very theoretical and limited in application, tending to symbol manipulation. It was omitted from Analytical Algebra II.

	AII.F.5: Describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative) with and without technology. Find the value of k given the graph of f(x) and the graph of f(x) + k, k f(x), f(kx), or f(x + k).	AA.LF.5: Describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative) with technology. Find the value of k given the graph of f(x) and the graph of f(x) + k, k f(x), f(kx), or f(x + k). Identify and analyze transformations within a real world context.	The Algebra II standard is addressed in its entirety. Although the standard is very theoretical and focused on the transformation of functions, the skill transfers nicely in a number of contexts. Because of this an emphasis on identifying transformations within real world contexts has been added.
SYSTEMS OF EQUATIONS	AII.SE.1: Solve a system of equations consisting of a linear equation and a quadratic equation in two variables algebraically and graphically with and without technology (e.g., find the points of intersection between the line y = –3x and the circle x^2 + y^2 = 3).	AA.LF.6: Solve a system of equations and/or inequalities consisting of a linear equation and a quadratic equation in two variables algebraically and graphically with technology (e.g., find the points of intersection between the line y = –3x and the parabola $y=x^2-6x-5$).	The Analytical Algebra II standard includes the addition of inequalities as well as a more appropriate example.
	AII.SE.2: Solve systems of two or three linear equations in two or three variables algebraically and using technology.		AII.SE.2 has been combined with AII.SE.3 to create AA.LF.7 because AII.SE.2 is only focused on theoretical problems.

	AII.SE.3: Represent real-world problems using a system of linear equations in three variables and solve such problems with and without technology. Interpret the solution and determine whether it is reasonable.	AA.LF.7: Represent real-world problems using a system of linear equations and/or inequalities in two or three variables. Solve such systems graphically or with matrices, as appropriate to the system, with technology. Interpret the solution and determine whether it is reasonable.	The Analytical Algebra II standard includes the addition of inequalities. The Analytical Algebra II standard includes the ability to solve with matrices. The Analytical Algebra II standard includes two or three variables as opposed to only three variables. This is due to the combination of AII.SE.2 and AII.SE.3 in to one standard, AA.LF.7.
QUADRATIC EQUATIONS AND FUNCTIONS	AII.Q.1: Represent real-world problems that can be modeled with quadratic functions using tables, graphs, and equations; translate fluently among these representations. Solve such problems with and without technology. Interpret the solutions and determine whether they are reasonable.	AA.QP.1: Represent real-world problems that can be modeled with quadratic functions using tables, graphs, and equations; translate fluently among these representations. Solve such problems with technology. Interpret the solutions and determine whether they are reasonable.	No changes.
	AII.Q.2: Use completing the square to rewrite quadratic functions into the form y = a(x + h)^2 + k, and graph these functions with and without technology. Identify intercepts, zeros, domain and range, and lines of symmetry. Understand the relationship between completing the square and the quadratic formula.	AA.QP.2: Rewrite quadratic functions into the form $y = a(x - h)^2 + k$ using a variety of strategies and graph these functions with technology. Understand that different forms of an equation can provide different information. Identify and interpret within a given context intercepts, zeros, domain and range, and lines of symmetry.	Removed the specification of completing the square as the only strategy to rewrite function. Analytical Algebra II emphasizes understanding the various forms of quadratics and the information they readily provide in each form. Analytical Algebra II added in the interpretation of important features of a

			quadratic function in given contexts.
	AII.Q.3: Use the discriminant to determine the number and type of solutions of a quadratic equation in one variable with real coefficients; find all solutions and write complex solutions in the form of a ± bi for real numbers a and b.	AA.QP.3: Use the discriminant to determine the number and type of solutions of a quadratic equation in one variable with real coefficients. Know there is an imaginary number, i, such that $i^2 = -1$, and understand the relationship to non-real complex roots.	Students no longer have to find the complex solutions of a quadratic function. Instead, students are asked to simply use the discriminant to determine the number and type of solutions. Students need to know the types of solutions possible and the relationship between the imaginary number and non-real complex roots. Students in Analytical Algebra II are not asked to find and write all solutions, including complex solutions.

EXPONENTIAL AND LOGARITHMIC EQUATIONS AND FUNCTIONS	AII.EL.1: Write arithmetic and geometric sequences both recursively and with an explicit formula; use them to model situations and translate between the two forms.	AA.LF.1: Model real world situations involving arithmetic sequences and understand that they can be defined both recursively and with an explicit formula. AA.EL.1: Model real world situations involving geometric sequences and understand that they can be defined both recursively and with an explicit formula.	Emphasis on real world situations and connections to linear functions (for arithmetic sequences) and exponential functions (for geometric sequences)
	AII.EL.2: Graph exponential functions with and without technology. Identify and describe features, such as intercepts, zeros, domain and range, and asymptotic and end behavior.	AA.EL.4: Graph exponential functions that model real-world situations with technology. Identify, describe, and interpret key features, such as intercepts, zeros, domain, range, asymptotic and end behavior.	Analytical Algebra II has an emphasis on exponential functions that model real world situations, as opposed to Algebra II, which is very theoretical and focuses on procedure, rather than application to the real world.
	AII.EL.3: Identify the percent rate of change in exponential functions written as equations, such as y = (1.02)^t, y = (0.97)^t, y = (1.01)12^t, y = (1.2)^t/10, and classify them as representing exponential growth or decay.	AA.EL.5: Given real-world contexts, identify the percent rate of change in exponential functions written as equations, such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{(12t)}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Analyze growth and decay using absolute and relative change and make comparisons using absolute and relative difference.	Analytical Algebra II has an emphasis on application to the real world. Addition of analysis of growth and decay in both absolute and relative terms. Addition of comparisons of growth and decay in both absolute and relative terms.

	AII.EL.4: Use the properties of exponents to transform expressions for exponential functions (e.g., the expression 1.15^t can be rewritten as (1.15^1/12)^12t ≈ 1.012^12t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%).	AA.EL.6: Use the properties of exponents to transform expressions for exponential functions in a given real-world context. (e.g., the expression 1.15^{t} can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%).	Analytical Algebra II has an emphasis on application to the real world.
	AII.EL.5: Know that the inverse of an exponential function is a logarithmic function. Represent exponential and logarithmic functions using graphing technology and describe their inverse relationship.	AA.EL.7: Know that the inverse of an exponential function is a logarithmic function. Represent exponential and logarithmic functions that model real-world situations using graphing technology and describe their inverse relationship. Use the inverse relationship between exponential functions and logarithms to evaluate expressions and solve equations in one variable.	Analytical Algebra II has and emphasis on application to the real world and multiple representations. The focus is on understanding these functions within a context. Combined AII.EL.5 and AII.EL.6 in to AA.EL.7.
	AII.EL.6: Use the laws of exponents to derive the laws of logarithms. Use the laws of logarithms and the inverse relationship between exponential functions and logarithms to evaluate expressions and solve equations in one variable.		Combined AII.EL.5 and AII.EL.6 in to AA.EL.7. Eliminated the use of laws of exponent to derive the laws of logarithms as it is very theoretical.

	AII.EL.7: Represent real-world problems using exponential equations in one or two variables and solve such problems with and without technology. Interpret the solutions and determine whether they are reasonable.	AA.EL.3: Represent real-world problems using exponential functions in one or two variables and solve such problems with technology. Interpret the solutions and determine whether they are reasonable.	No change
POLYNOMIAL, RATIONAL, AND OTHER EQUATIONS AND FUNCTIONS	AII.PR.1: Solve real-world and other mathematical problems involving polynomial equations with and without technology. Interpret the solutions and determine whether the solutions are reasonable.	AA.QP.4: Represent real-world problems that can be modeled with polynomial functions using graphs and equations. Solve such problems with technology. Interpret the solutions and determine whether they are reasonable.	Added the requirement that students be able to represent real-world problems that can be modeled with polynomial functions using both graphs and equations. The focus should be on using the graph to answer questions within a given context.
	AII.PR.2: Graph relations and functions including polynomial, square root, and piecewise-defined functions (including step functions and absolute value functions) with and without technology. Identify and describe features, such as intercepts, zeros, domain and range, end behavior, and lines of symmetry.	AA.QP.5: Graph polynomial functions that model a real-world situation with technology. Identify, describe, and interpret key features in the context of the situation, such as intercepts, zeros, domain and range, end behavior, maxima and minima, and lines of symmetry.	Excluded relations. This standard was kept specifically to polynomial functions. Square root, and piecewise defined functions were separated from this standard and moved to AA.R.1.

	AII.PR.3: Solve real-world and other mathematical problems involving rational and radical functions, including direct, inverse, and joint variation. Give examples showing how extraneous solutions may arise.	AA.R.1: Represent and solve real-world problems that can be modeled with rational functions (including direct, inverse, and joint variation) using tables, graphs, and equations; translate among these representations. Graph rational functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior. AA.R.2: Represent and solve real-world problems that can be modeled with radical functions using tables, graphs, and equations; translate among these representations. Graph radical functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior. AA.R.3: Represent and solve real-world problems that can be modeled with piecewise-defined functions (including step functions and absolute value functions) using tables, graphs, and equations; translate among these representations. Graph piecewise-defined functions with technology. Identify, describe, and interpret features, such as intercepts, zeros, asymptotes, domain and range, and end behavior.	Emphasis on real world problems. The addition of representing rational functions, not just solving them, so that the representation (graph) can be used in the analysis. The addition of representing radical functions, not just solving them The addition of representing piecewise defined functions, not just solving them The identification and describe piece from AII.PR.2 was combined here as well. Each function family was separated out in to its own standard as to support the importance of each one.

DATA ANALYSIS, STATISTICS, AND PROBABILITY	AII.DSP.1: Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	AA.DSP.1: Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization and possible sources of bias relate to each.	Addition of possible sources of bias in sample surveys, experiments, and observational studies.
	AII.DSP.2: Use technology to find a linear, quadratic, or exponential function that models a relationship for a bivariate data set to make predictions; compute (using technology) and interpret the correlation coefficient.	AA.DSP.2: Choose, create, and critique, with technology, mathematical models (linear, quadratic and exponential) for bivariate data sets. Use the models to interpolate and/or extrapolate, to answer questions, and to draw conclusions or make decisions, addressing limitations and long-term ramifications. Recognize when a change in model is needed. Interpret the	Language calls out the importance of choosing, creating, and critiquing a model with technology, not just to find a model with technology. Changed language from making predictions to interpolating and/or extrapolating with the model, answer questions, draw conclusions or make decisions.

		correlation coefficient for linear models.	Addition of addressing limitations and long-term ramifications of models. Addition of recognition of the need for a change in model. Language surrounding correlation coefficient was made specific to linear models.
	AII.DSP.3: Organize, graph (e.g., line plots and box plots), and compare univariate data of two or more different data sets using measures of center (mean and median) and spread (range, interquartile range, standard deviation, percentiles, and variance). Understand the effects of outliers on the statistical summary of the data.	AA.DSP.4: Analyze and compare univariate data of two or more different data sets using measures of center (mean, median, and mode), shape, and spread (range, interquartile range, standard deviation, percentiles, and variance) making use of technology. Understand the effects of outliers on the statistical summary of the data.	Removed the limitation of univariate data being displayed as line plots and box plots. Language changed from organize, graph, and compare to analyze and compare. Emphasis is taken of the student needing to graph the data and put on the analysis of the data and comparison to other sets of data.
	AII.DSP.4: Record multiple observations (or simulated samples) of random events and construct empirical models of the probability distributions. Construct a theoretical model and apply the law of large numbers to show the relationship between the two models.	AA.DSP.5: Record multiple observations (or simulated samples) of random events and construct empirical models of the probability distributions. Construct a theoretical model and apply the law of large numbers to show the relationship between the two models.	No change

	AII.DSP.5: Understand dependent and independent events, and conditional probability; apply these concepts to calculate probabilities.	AA.DSP.6: Evaluate the validity of claims based on empirical probabilities and theoretical probabilities, including those derived from dependent and independent events. Draw conclusions and make decisions in various probabilistic contexts. Make use of different representations of data including two-way tables and tree diagrams.	Added language to evaluate the validity of claims based on empirical and theoretical probabilities. Added language to draw conclusions and make decisions in various probabilistic contexts. Added language to make use of various representations of data, including two-way tables and tree diagrams.
	AII.DSP.6: Understand the multiplication counting principle, permutations, and combinations; apply these concepts to calculate probabilities.	AA.DSP.7: Determine the nature and number of elements in a finite sample space to model the outcomes of real-world events using the multiplication counting principle, permutations, and combinations.	Language added to emphasize the determination and nature of elements in a finite sample space to model the outcomes of real world events.
		AA.DSP.3: Read, interpret, and make decisions about data summarized numerically using measures of center and spread, in tables, and in graphical displays (line graphs, bar graphs, scatter plots, and histograms), e.g., explain why the mean may not represent a typical salary; critique a graphical display by recognizing that the choice of scale can distort information.	Students are not asked to create the data displays but rather to focus on reading, interpreting and making decisions about data summarized numerically in tables and in graphical displays

Proposed Professional Development Plan
Analytical Algebra II
11/20/2018

Rationale

Analytical Algebra II focuses on the application of mathematics in various disciplines including business, finance, science, career and technical education, and social sciences. This course covers most of the traditional Algebra II standards, but the focus is on the application of algebraic concepts rather than theoretical concepts. Additionally, data analysis, statistics, and probability content should be included throughout the course, as students collect and use univariate and bivariate data to create and interpret mathematical models.

The vision for Analytical Algebra II is to develop the capacity of students to become quantitatively literate global citizens. Much of the information available is quantitative in nature and we must equip our students with the skills to analyze and process this data to make every day decisions. For this reason, we have used language from the Quantitative Reasoning course to reframe the traditional Algebra II standards in order to emphasize analysis and interpretation in real-world situations.

A professional development plan has been developed to assist teachers in a shift in pedagogical practice that is critical in the success of the Analytical Algebra II course.

Proposed Work

<u>Implementation Year 1:</u>
Spring 2019

- 1-day regional training for course instructors provided by DOE
    - Deepen understanding of the creation/intent of course
    - Develop initial understanding of crosswalk between Algebra II and Analytical Algebra II
    - Build consistency across schools about expectations for student entry and success in course
    - Develop initial consensus across schools in regards to instructional practice and implementation of standards
    - Develop curricular calendar and unpack standards

Summer 2019

- 2-day training (or two 1-day trainings) for course instructors provided by DOE
    - Build capacity of instructors to make a pedagogical shift in content delivery

- Develop exemplar lessons to be shared across community of practice
- Develop initial stability in assessment grading practices (develop rubric?)

Academic year 2019-2020

- In Service – 1-day service training each semester (fall and spring)
  - Address successes/concerns
  - Continue to build capacity of instructor's pedagogical shift in content delivery
  - Continue to develop exemplar lessons to be shared across community of practice
- Communities of Practice –Teachers will have the opportunity to meet in small groups of 10-15 individuals each twice per semester (virtually?) with DOE staff member
  - Guided questions
- Development of Moodle (?) "course"
  - Begin development of "warehouse" for exemplar lessons and other course related materials

<u>Implementation Year 2:</u>
Summer 2020

- 2-day training (or two 1-day trainings) for new course instructors provided by DOE and teacher leaders (?)
  - Deepen understanding of the creation/intent of course
  - Develop initial understanding of crosswalk between Algebra II and Analytical Algebra II
  - Build consistency across schools about expectations for student entry and success in course
  - Develop initial consensus across schools in regards to instructional practice and implementation of standards
  - Build capacity of instructors to make a pedagogical shift in content delivery
  - Develop exemplar lessons to be shared across community of practice
  - Develop initial stability in assessment grading practices (develop rubric?)

Academic year 2020-2021

- In Service – 1-day service training each semester (fall and spring)
  - Continue to build capacity of instructor's pedagogical shift in content delivery
  - Continue to develop exemplar lessons to be shared across community of practice

- Communities of Practice –Teachers will have the opportunity to meet in small groups of 10-15 individuals each twice per semester (virtually?) with DOE staff member
    - Guided questions