Standard on Ratio Studies

Approved July 1999

International Association of Assessing Officers

This standard revises and replaces the 1990 *Standard on Ratio Studies.*

The assessment standards set forth herein represent a consensus in the assessing profession and have been adopted by the Executive Board of the International Association of Assessing Officers. The objective of these standards is to provide a systematic means by which concerned assessing officers can improve and standardize the operation of their offices. The standards presented here are advisory in nature and the use of, or compliance with, such standards is purely voluntary. If any portion of these standards is found to be in conflict with the *Uniform Standards of Professional Appraisal Practice* (*USPAP*) or state laws, *USPAP* and state laws shall govern.

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Standard on Ratio Studies

1. Scope

This standard provides recommendations on the design, preparation, interpretation, and use of ratio studies for equalization, the evaluation of appraisal performance, and the quality control operations of an assessor's office.

The standard is divided into fifteen sections; the last is a glossary of key terms. Depending on one's interest, one can either read the standard in its entirety or focus on individual sections. Many topics addressed in the standard are discussed in more detail in IAAO (1990, chapter 20) and Gloudemans (1999, chapter 5).

In this standard, "ratio study" is used as a generic term for all studies designed to evaluate appraisal performance or determine taxable value through a comparison of appraised or assessed values estimated for tax purposes with independent estimates of market value based on either sales prices or independent appraisals. The term is used in preference to the term "assessment ratio study" because use of assessments in such studies can mask the true level of appraisal and confuse the measurement of appraisal uniformity when the legal assessment level is other than 100 percent for all property.

2. Introduction

2.1 The Concepts of Market Value and Ad Valorem Appraisal

Market value is the most probable selling price of a property in terms of money in an open-market, arm's-length transfer between a willing and well-informed buyer and willing and well-informed seller, neither under duress. The major responsibility of assessing officers is estimating the market value of properties based on statutory requirements. The viability of the property tax depends largely on the accuracy of such estimates, because they are the legal basis of the assessed values from which property tax bills are calculated. The accuracy of appraisals made for assessment purposes is, therefore, of concern, not only to assessors, but also to taxing authorities, property owners, and their elected representatives. Ratio studies provide a means for evaluating the accuracy of those appraisals.

Appraisal accuracy refers to the degree to which properties are appraised at market value, as defined by professional standards (see section 14), state and provincial statutes, and administrative codes. A ratio study compares appraisals with indicators of market value (either sales or independent appraisals). The ratios used in such studies are formed by dividing the former by the latter. When the statutory level of assessment is 100 percent, assessed values before subtraction of any exemptions (for example, homestead exemptions) may be used in place of appraisals because they are, in effect, one and the same. When statutes mandate different assessment levels for different classes of property, appraised values are preferred, because they simplify comparisons between legal classes and avoid interpretive problems when parcels from different classes are combined.

Market values cannot be observed directly. They are represented in ratio studies either by sales prices, screened and adjusted as necessary (see section 6), or by independent appraisals. Sales prices provide the only objective estimates of market values and under normal circumstances should provide good surrogates of market value. Independent appraisals are appropriate when sales are insufficient or when statutes require that properties be appraised for tax purposes on other than a market value basis (see section 9). If statutory constraints require appraisal at other than market value, it is often appropriate to apply these constraints to the value indicator used in the ratio study to test the appraisal.

2.2 Aspects of Appraisal Performance

There are two major aspects of appraisal accuracy: level and uniformity. Appraisal level refers to the overall ratio of appraised values to market values. The level provides information about the degree to which legal requirements are met. Uniformity refers to the degree to which different properties are appraised at equal percentages of market value, that is, the degree to which property tax levies (aside from statutory differences in assessment levels) are distributed in proportion to market value. Section 7 discusses measures of appraisal level and uniformity.

A ratio study may examine the overall degree of accuracy within and among groups of properties (for example, type of property, neighborhood, or age group), the overall degree of accuracy within a local assessment district, and the degree of accuracy of an entire province or state.

2.3 Uses of Ratio Studies

The uses of ratio studies can be as wide-ranging as concerns about appraisal accuracy. Some key uses of ratio studies are discussed below. Chapter 5 of *Mass Appraisal of Real Property* (Gloudemans 1999) contains a more detailed discussion of the uses and applications of ratio studies.

2.3.1 Monitoring Appraisal Performance

2.3.1.1 Use by Assessors

Both state/provincial and local assessment agencies use ratio studies to monitor appraisal performance, to determine the need for a general reappraisal, to establish

priorities for reappraisal of selected groups of properties, to identify potential problems with appraisal procedures, to conduct market analyses, and to adjust appraised values between reappraisals.

2.3.1.2 Use by Oversight Agencies

State/provincial agencies often perform ratio studies to advise assessors and the public about local appraisal conditions and to order reappraisals or equalize locally determined assessments.

2.3.2 Equalization

Oversight agencies may use ratio studies to equalize, directly or indirectly, appraisals or assessments in taxing jurisdictions. Direct equalization is accomplished by adjusting appraisals within jurisdictions or property classes and may involve adjusting appraisals of centrally assessed properties. When indirect equalization is used, appraisals are not adjusted. Instead, indirect equalization involves computing hypothetical values that represent the oversight agency's best estimate of taxable value, given the statutorily required level of assessment or market value. Indirect equalization ensures proper distribution of intergovernmental transfer payments between state or provincial and local governments despite different levels of appraisal between jurisdictions or property classes.

2.3.2.1 Direct Equalization

Many states and provinces have authority and specific procedures for direct equalization (Dornfest 1997), which involves converting ratio study results into adjustment factors (trends) and ordering locally determined appraised or assessed values to be changed to more nearly reflect market value or the statutorily required level of assessment. Direct equalization has an advantage in that it can be applied to specified strata, such as property classes, geographic areas, and political subdivisions, that fail to meet appraisal level performance standards (Dornfest 1997). Direct equalization also produces results that are more visible to the taxpayer and will more clearly reduce perceived inequities between classes (*Standard on Property Tax Policy* [IAAO 1997]). For example, direct equalization ensures proper and equal application of debt and tax rate limits and equal effect of partial exemptions.

Direct equalization involves use of adjustment factors, which produce effects very similar to those produced by application of "trending" or "index" factors that are commonly used for value updating by local assessing jurisdictions. The most significant differences typically are the level of the jurisdiction originating the adjustments and the stratification of property to which the factors are applied. Local jurisdictions with primary assessment responsibility may develop factors as part of an ongoing reappraisal program. Such factors commonly are applied to properties by property type, location, size, age and other characteristics (IAAO 1990, p. 310). It is rare for equalization factors developed by oversight agencies to be applied to strata more specific than property class or broad geographic areas. Often such factors are applied jurisdictionwide.

States and provinces that employ direct equalization techniques should understand that such equalization is not a substitute for appraisal or reappraisal. Although direct equalization will improve *inter-stratum* equality in effective tax rates and will lessen the effect of assessment practices that improperly favor one stratum over another, such equalization cannot improve *intra-stratum* uniformity. For this reason, reappraisal orders should be considered as the primary corrective tool for uniformity problems, and direct equalization should be considered appropriate only if time constraints preclude such an approach. Indirect equalization may still be considered when appraisal uniformity conditions preclude direct equalization.

2.3.2.2 Indirect Equalization

The most common use of indirect equalization is to ensure proper funding distribution, particularly for school districts. Such equalization provides an estimation of the proper tax base (acknowledging statutory constraints such as agricultural use value) despite appraisals that may be higher or lower than statutorily required levels in certain jurisdictions. For example, if the assessed value of residential property in a jurisdiction is $750 million, but a residential ratio study shows an assessment level of 75 percent, while the legally required level of assessment is 100 percent, an equalized value of $1,000 million could be computed ($750 million/0.75). This adjusted or equalized value would then be used to apportion payments between states or provinces and local governments.

Indirect equalization results in fairer funding apportionment because the overall appraisal levels of the taxing jurisdictions tend to vary. If there were no equalization, the extent that a jurisdiction under- or overestimated its total tax base would result in over- or underapportionment of funds. Indirect equalization does not correct for under- or overappraisal between classes of property within a jurisdiction, is less visible to taxpayers, and often lacks checks and balances associated with direct equalization (*Standard on Property Tax Policy* [IAAO 1997]). However, by adjusting governmental payments, or tax rates, or partial exemptions, indirect equalization does tend to encourage taxing jurisdictions to keep their overall tax bases close to the required level.

Whether used to equalize shared funding or tax rates, the degree of equalization of the property tax is more limited than with direct equalization. This is so because indirect equalization generally is applied to or affects only a portion of the funding or property tax levy (perhaps the school general levy or city levy). Since individual property values are not adjusted, the uniformity related constraints discussed under direct equalization are not as critical. How-

ever, indirect equalization usually is applied to the jurisdiction, rather than a stratum, and therefore resolves only interjurisdictional discrepancies in assessment level. In addition, more properties in strata with poor uniformity will be affected disproportionately. For this reason, indirect equalization also is not a substitute for reappraisal.

2.3.3 Use of Ratio Studies in Appeals

Assessors, appeal boards, taxpayers, and taxing authorities can use ratio studies to evaluate the fairness of funding distributions, the merits of class action claims, or the degree of discrimination. However, ratio study statistics cannot be used to judge the level of appraisal of *individual* parcels. Such statistics may be used to adjust assessments on appealed properties to the common level within the appropriate stratum, provided that such level is outside of the range of acceptability specified in section 14.1. For example, if it is proven by an appraisal that the property under appeal is valued at 10 percent over market value, but the stratum is provably at no more than 80 percent of market value, the assessment of the appealed property could be reduced to 80 percent of market value.

2.4 Legal Aspects of Ratio Studies

Property taxation is governed by federal, state, and provincial constitutions and statutes, many of which require uniform treatment of property taxpayers. Ratio studies play an important role in judging whether uniformity requirements are met. Relevant Canadian federal statutes based on the Constitution Acts of 1867–1975 provide that municipal councils cannot discriminate between taxpayers of the same class within municipalities.

Relevant United States federal provisions include the Bill of Rights, the commerce clause of the United States Constitution, the Fourteenth Amendment, and the Tax Injunction Act (28 U.S.C. § 1341). Together they guarantee basic protections and due process while still granting states the authority to classify property and grant reasonable exemptions. Many constitutions have clauses that require uniformity in the assessment and taxation of property, although some jurisdictions, either by constitution or statute, permit certain differences between classes. Ratio studies provide a gauge of whether uniformity requirements are being met.

A key U.S. federal statute relating to ratio studies is the U.S. Railroad Revitalization and Regulatory Reform Act ("4-R Act") of 1976 (49 U.S.C. § 11501). The 4-R Act requires that rail transportation property be assessed for tax purposes at no more than 105 percent of the assessment level of other commercial and industrial property in the same taxing jurisdiction. Similar federal statutes relate to air transportation property, motor carriers, and bus lines (49 U.S.C. §§14502 and 40116).

The 4-R Act provides that ratio studies be used to measure alleged discrimination. In such cases, as in any ratio study, the purpose of the study must be clearly defined and the study must be conducted so that it accurately evaluates the issues at hand. Important issues in ratio studies conducted pursuant to the 4-R Act include the proper definition of "other" commercial and industrial property, screening and adjustments to sales data, proper measures of the level of appraisal, and the combining and weighting of centrally valued and locally assessed properties.

3. Design Considerations

In considering the design of any ratio study, analysts should take into account the intended use or purpose of the study. The most important consideration is that the study sample be representative of the population of properties (direct equalization) or the population of dollars of property value (indirect equalization) in terms of the level and uniformity of appraisal (see section 5.5 and 5.5.1). The study should achieve the appropriate level of sophistication and detail, employ valid sampling methods, use appropriate parametric and distribution-free (nonparametric) statistics (see section 7.8.1), and ensure that the data used in the study are accurate and representative.

3.1 Level of Sophistication and Detail

A basic design principle is to keep the study as simple as possible consistent with its purpose. Ratio studies are not all alike and should be tailored to a particular purpose.

Ratio studies that are intended primarily for estimation of the total property value available for taxation in a taxing jurisdiction require a substantially different design than ratio studies intended for estimation of the general level and uniformity of appraisal. For instance, a ratio study intended for market value estimation (indirect equalization) requires samples that are representative of the distribution of property *value* in the jurisdiction, whereas a ratio study intended for estimation of the general level and uniformity of appraisal (direct equalization or monitoring appraisal performance) requires a sample that is representative of the distribution of *properties* in the jurisdiction. The former study is designed to determine the total value of property in the jurisdiction; the latter is designed to determine how individual properties are treated in regard to level and uniformity. See section 5 for more information about sample selection.

The statistical sophistication required in a study varies greatly, depending on its intended use. Increased computerization has made generation of accurate, sophisticated statistics virtually automatic. Although every study does not require the same level of statistical detail, practitioners should ensure that each ratio study includes measures of appraisal level, appraisal uniformity, and statistical reliability, as well as graphs, charts, or other pictorial representations showing distributions and patterns in the data. In ratio studies designed for equaliza-

tion or to evaluate equity between property groups, various statistical tests of hypotheses concerning assessment equity are appropriate. Ratio study practitioners should prepare explanatory materials or provide training to enable the user to understand the results of these analyses.

Finally, the design of a ratio study always requires decisions that weigh the value of greater statistical reliability against available resources and costs. Consequently, there is no model ratio study design that will serve all jurisdictions or all situations equally well. Informed, reasoned judgment and common sense are required in the design of ratio studies.

3.2 Sampling

Whatever the degree of statistical sophistication, a ratio study is a form of applied statistics, because the analyst draws conclusions about the appraisal of the universe (population) of properties based only on those that have sold during a given time period or that have been independently appraised. The sales or independent appraisals constitute the sample.

To determine the accuracy of appraisals with absolute certainty, it would be necessary for all properties in the population to have been sold in arm's-length, open-market transfers near the appraisal date. Since this is not the case, ratio studies must use samples and draw inferences or conclusions about the population from these samples.

As an example, a ratio study might consist of twenty sales from a particular neighborhood containing several hundred properties. If the sample is representative of the properties in the neighborhood, valid conclusions can be made about the overall accuracy of appraisals in the neighborhood, which is deemed to be the population in this case. (Sections 5.5 and 8 address issues of validity and representativeness.)

3.3 Data Accuracy

The findings of a ratio study can only be as accurate as the data used in the study. No matter what the purpose of the study, the data must be as accurate as possible. Personnel involved in collecting, confirming, screening, and adjusting sales data or making appraisals should be familiar with real estate conveyance practices in their region, proficient in the principles and practices of real estate appraisal, and aware of current real estate markets. Accuracy of data entered into or transferred between computer systems should be ensured (see section 13.3).

4. Steps in Ratio Studies

Ratio studies generally involve six basic steps: (1) definition of purpose and objectives, (2) collection and preparation of market data, (3) matching appraisal and market data, (4) stratification, (5) statistical analysis, and (6) evaluation and use of results.

4.1 Definition of Purpose and Objectives

The first step in any ratio study is to determine and state clearly the study's purpose and objectives. This crucial step determines the scope, content, depth, and required flexibility of the study. A study designed for one purpose may not produce results that are reliable for another purpose. Objectives of ratio studies include equalization of values, estimation of total market values for purposes of funding distribution, evaluation of the level and uniformity of appraisals, identification of appraisal priorities, and determination of whether administrative standards have been met.

4.2 Collection and Preparation of Market Data

The reliability of a ratio study depends on how well the sales or independent appraisals used in the study reflect market values. Sales data should be verified, edited, and adjusted as necessary for financing, personal property, and time of sale (see section 6). The underlying principle for review of data is to ensure that the sample is adequate in size (see section 8.3), but at the same time to exclude sales that provide invalid indicators of market value. Independent appraisals used in ratio studies must employ valid appraisal techniques, must comply with the appropriate sections of the *Uniform Standards of Professional Appraisal Practice (USPAP)*, and must reflect market values for the date being studied.

4.3 Matching Appraisal Data and Market Data

The basic physical characteristics of each property used in the ratio study must be the same when appraised for tax purposes and when sold or appraised by the review agency. For sales this implies two essential steps. First, one should ensure that the legal descriptions match. If a parcel is split after the appraisal date, a sale of any of its parts should not be used in the ratio study. A sale of two or more adjacent parcels may be used by summing the appraised values of the individual parcels, provided that the combined parcels would have a market value equal to this sum. Proper matching is easier if parcels are renumbered when split or combined (see *Standard on Cadastral Maps and Parcel Identifiers*, section 9 [IAAO 1988] and IAAO [1990, 464]). If they are not or if parcel numbers reported on real estate transfer documents are in error, legal descriptions must be examined to ensure that parcel numbers are correct.

Second, once the legal descriptions match, one must ensure that the rights transferred, the permitted use, and the physical characteristics are the same at time of assessment and time of sale. In the case of parcels with new improvements, for example, it is important to avoid matching sales prices of vacant lots against appraised values that include new construction, although such sales could be used in a ratio study for vacant land by using the appraised value of the land only. Besides new improvements, parcels should be checked for significant physical additions, renovations, or demolitions between the dates of assessment and sale. In general, parcels with

such changes should be excluded from ratio studies. However, minor physical changes should not cause a parcel to be excluded.

4.4 Stratification

Stratification divides all the properties in a jurisdiction that fall within the scope of the study into two or more subpopulations (strata). Stratification provides a more complete and detailed picture of the extent and nature of appraisal performance and can enhance sample representativeness. Stratification, therefore, is essential in most ratio studies conducted by local assessors and is highly desirable, if not essential, in studies used for equalization and for similar purposes.

Strata should be chosen with attention to the purpose of the ratio study, legal requirements, real estate submarkets, available sales (or appraisal resources in the absence of sales), and reliability requirements. Each type of property subject to a distinct level of assessment should constitute a stratum. Neighborhoods and other property groups or strata can also be developed, provided that such strata yield sufficient data. Value stratification, which may implicitly incorporate such factors as age and construction type, is appropriate for indirect equalization, but can be misleading if assessment dispersion is high or if the results are used to gauge vertical equity (see section 14.2.7).

When the purpose of the study is to monitor assessment operations, particularly as an internal control for the assessor, flexibility in stratification is essential. In such cases, it is also highly desirable to be able to stratify on the basis of more than one characteristic.

Statistical issues in the determination of strata include the size of the population and resulting strata and the likely variability of the ratios in each stratum (see section 5.5). Care must be taken *not* to overstratify, that is, to create strata that are too small to achieve statistical reliability (see section 8 and Sherrill and Whorton [1991]). No conclusion about stratum level or uniformity should be made from stratum samples that are unreliably small (resulting in unacceptably large margins of error). On the other hand, if small margins of error for a stratum are not an issue, as in studies made primarily to determine the total taxable value within a jurisdiction, a reasonable number of strata with small samples can increase representativeness and may reduce the margin of error for the overall jurisdictionwide sample. Ultimately, the degree of stratification will be determined largely by available sales data, unless it is cost-effective and practical to add sufficient independent appraisals. If sufficient sales or appraisals are not available for a given stratum, it should be combined with similar strata. When strata are combined, provided there is no reason to suspect dissimilar ratios as evidenced by different level or uniformity measures, such combination will permit broader applicability of ratio study results and prevent ratio study analysis from becoming too focused on substrata with few sales or appraisals.

4.4.1 Timing of Stratification for Equalization Studies

Oversight agencies generally should define the strata prior to acquiring and compiling data for the ratio study. This will enhance cooperation between the oversight agency and the jurisdiction appraising the property subject to equalization. Once strata are established for equalization purposes (especially in the case of direct equalization), an oversight agency should not redefine them without reasonable advance notice to the assessing jurisdiction.

4.4.2 Stratification for Equalization of Funding Distributions

Often the goal of the ratio study is to equalize funds being transferred between states or provinces and local units of government on the basis of market value. Such studies can use a substantially different approach to stratification than ratio studies intended for estimation of the general level and uniformity of appraisal. The purpose of stratification is to minimize distortion due to different assessment levels. In any case, chosen strata must be represented in the ratio study according to their proportional share of value in the population (see section 5.5.1).

In ratio studies conducted to equalize distribution of funds, overstratification can create particular problems. For example, in a large school district, a more representative sample could be created by developing many class and location strata. However, in a small, adjacent school district the use of these same strata may produce unreliably small sales samples. Options for preventing this problem include addition of independent appraisals to provide constant margins of sampling error, use of different strata in small and large districts, and use of fewer strata consistently regardless of district size. The option chosen should maximize sample representativeness and reliability at the level of aggregation from which conclusions will be drawn. For instance, a taxing jurisdiction sample could be stratified by major property type and by value within property type. If conclusions are drawn by property type and for the taxing jurisdiction as a whole, then sample reliability should be maximized for these groupings, but need not be maximized at the value substratum level. If two strata from which conclusions will be drawn do not differ appreciably in assessment level or uniformity measures, it is usually better to combine the strata.

4.5 Statistical Analysis

A ratio should be computed for each parcel in the study, and measures of appraisal level, uniformity, and reliability should be calculated for the entire jurisdiction and each stratum. Graphs and charts are often useful for illustrating results. When ratio studies are conducted for equalization purposes, confidence intervals and statisti-

cal tests can be used to determine whether one can conclude at a given confidence level that appraisal performance meets or falls outside of mandated standards. Without such measures of reliability, the sample statistics concerning level of appraisal should not be considered conclusive. Section 7 discusses ratio study statistics and analyses.

4.6 Evaluation and Use of Results

A properly designed ratio study is a powerful tool for analyzing appraisal performance and suggesting strategies for improvement. For example, an assessor who conducts studies by type of property and neighborhood can direct appraisal resources to those property groups in greatest need. Section 2.3, IAAO (1990, chapter 20), and Gloudemans (1999, chapter 5) suggest some of the many other uses of ratio studies.

Users of ratio studies, however, should recognize the inherent limitations of such studies. The following should be kept in mind:

1. A ratio study cannot provide perfect information about appraisal performance. Lack of sufficient sales or overrepresentation of one area or type of property due, for example, to a highly active market can distort results.

2. Ratio study validity requires that sold or independently appraised parcels be appraised with the same frequency, at the same percent of market value, and in the same manner as unsampled parcels. Violation of this condition seriously undermines the validity of any ratio study by reducing representativeness of the study and applicability of the results. When the purpose of the study is equalization, lack of independence will subvert attempts to improve equity (direct equalization) and result in incorrect distribution of funds between states or provinces and local jurisdictions (indirect equalization). To guard against these possibilities, assessing officials should ensure that sold and unsold properties are similarly appraised and take remedial measures where they are not (see section 10).

3. Findings should be used only in ways that are consistent with the purpose(s) for which the study was designed. It is inappropriate, for example, to use a ratio study to determine the market value of an individual property during the appeal process. Although ratio studies often are used to develop trending factors to adjust strata, it is inappropriate to trend values indefinitely in place of a reappraisal (see section 2.3.2).

4. Judgment is essential when conducting a ratio study or when evaluating or using the results. Ratio studies reduce uncertainty about appraisal accuracy by providing an objective basis for evaluating appraisal level and uniformity. Nevertheless, real estate markets consist of many individual properties, each unique in some way, and market participants who are imperfectly informed and not always rational. This, together with the statistical errors inherent in any sampling process, makes judgment essential when evaluating a ratio study and acting on the results.

5. Timing and Sample Selection

5.1 Data Requirements and Availability

Data requirements and availability must be evaluated early in the design of a ratio study. The purpose of the study will dictate certain data requirements. The availability of data will, in turn, influence design of the study and may call for revisions in the objectives of the study, limit the usefulness of the calculated statistics, or both. The information generally required for a ratio study includes the nature and distribution of the population, assessment information, indicators of market value, and property characteristics.

5.1.1 Nature of the Population

It is essential to know the type of properties, market conditions, and composition of the population in terms of age, size, value range, and so forth. Such information is needed to make informed decisions in designing the study and interpreting the results. Very large properties that rarely sell (for example, a large power plant) can be ignored in a ratio study designed to monitor local assessment performance, but must be considered in ratio studies designed to estimate market value for funding distribution.

5.1.2 Assessment Information

Appraised or assessed values are the numerators in the ratios used in a ratio study. Information about appraisal dates, legal requirements concerning reappraisals, the dates on which the appraisals were originally set, and the period they remained in effect are required for establishing the date of analysis (see section 5.3) and the period from which sales data will be drawn (see section 5.4).

5.1.3 Indicators of Market Value

Indicators of market value, either sales or independent appraisals, are the denominators in the ratios. Limitations in the availability and integrity of such data are important determinants of the design and usefulness of a ratio study. Specific information about the date, amount, terms, and conditions of sale is required for proper sales analysis. Appraisals used in ratio studies must employ sound methods and techniques and provide accurate indicators of market value (see section 9).

5.1.4 Property Characteristics

Information on property characteristics is crucial for determining whether a property as it was assessed corresponds to the property as it was sold or appraised (see section 4.3). Knowledge of key property characteristics is also essential for effective stratification (see section

4.4). In addition, the inclusion of property characteristics will improve the usefulness of reports that list information about the sales used in ratio studies.

5.2 Frequency of Ratio Studies

The purpose of a ratio study dictates how often it should be conducted. Regardless of the reappraisal or equalization cycle, ratio studies made by assessors as an internal control procedure and by property tax supervisory and equalization agencies should be conducted at least annually. This enables potential problems to be recognized and corrected before they become serious, as might happen if ratio studies were conducted only in tandem with appraisal cycles.

When there is a revaluation, assessors should (if possible) conduct at least three ratio studies: one based on preliminary values so that any major deficiency, such as lack of uniformity between neighborhoods, can be corrected; a second based on values used in assessment notices; and a third based on final values after completion of the first, informal phase of the appeals process. The final study can be used in planning for the following year. Where possible, ratio studies conducted by state- or provincial-level equalization agencies should use final values established at the local level, inclusive of changes made by local appeal boards up to that point in time. However, if local appraisers or boards "chase sales," or set values in a manner that is dissimilar to the way other property values have been set, the sample may not be representative and cannot be used without investigation (see section 10).

In addition, ratio studies are often conducted ad hoc to evaluate appraisal procedures, a discrimination complaint, or other specific questions. Ratio studies should be designed with sufficient flexibility to accommodate such occurrences.

5.3 Date of Analysis

Because the purpose of the ratio study is to evaluate the relationship between appraisals (or assessments) and market values at a specific time, a specific date of analysis should be selected for the study. This date will depend on the purpose of the study, but generally is the assessment date of the tax year being studied, which may be the current, the next, or the past year. The current year can be used, provided that the sample has been thoroughly tested for sales chasing. The assessment date of the next tax year should be used when the purpose of the study is to evaluate preliminary values in a reappraisal. Conversely, the date of analysis will be a past year when appraisals from past years are being evaluated or to avoid the effects of sales chasing. When prior year assessments are used to gauge current performance (to avoid sales chasing), the results should be adjusted for any reappraisal activity or assessment changes that occurred in the population (net of new construction) between the prior and current years.

5.4 Period from Which Sales Are Drawn

This period will depend on the purpose of the study and on sales activity, although the study period is sometimes set by statute or administrative rule. In general, the period should be as short as possible and, ideally, no longer than one year. Often, however, a longer period is required to produce an adequate sample for one or more strata within a jurisdiction. The period selected for each stratum can vary, although this may create practical difficulties and inconsistencies if sales prices are not adjusted for time.

The sales period will also vary with the intended use of the study. If the purpose of the study is equalization, using sales *after* the appraisal date (adjusted for time as necessary) helps ensure the independence of appraisals and sales prices. (Use of prior years' appraised values also helps ensure this independence.) A sales period spanning the appraisal date can be used if measures are taken to ensure the independence of appraisals made after the earlier sales. This approach has the advantage of reducing the importance of time adjustments, although such adjustments should still be made if markets have changed significantly over the period in question. At other times, of necessity, the sales period will lie before the appraisal date, for example, when preliminary values are being evaluated during a reappraisal.

In order to secure an adequate sample, sales used in ratio studies can span a period of as long as five years, provided there have been no major economic shifts and sales prices have been adjusted for time as necessary. Also, if a prior revaluation resulted in major changes in property taxes, sales before the revaluation should not be used without applying any required time adjustments to sales prices to account for the capitalization of tax shifts. (See IAAO [1990, appendix 5-3] and Gloudemans (1999, chapter 6) for a discussion of time-adjustment methods.)

5.5 Representativeness of Samples

In general, a ratio study is valid to the extent that the sample is *representative* of the population. In many kinds of statistical studies, samples are selected randomly from the population to ensure representativeness. Ratio study samples based on independent appraisals can be randomly selected. Because sales do not represent true random samples, extra care must be taken to ensure representativeness. To the extent that any ratio study is based on a representative sample, the statistical measures computed from the study are valid. A ratio study sample is considered representative when the distribution of ratios of properties in the sample reflect the distribution of ratios in the population. Because the distribution of ratios in the population cannot be directly ascertained and appraisal accuracy may vary from property to property, depending on property type and characteristics, representativeness can best be achieved by selecting a sample that proportionately reflects salient value-related property characteristics.

A property should be included in a sample based on characteristics of the property and not actions or characteristics of the owner. In reviewing the representativeness of a sample, one should determine whether the sample proportionately reflects ratio-related property characteristics of the population of sold and unsold properties.

5.5.1 Achieving Representativeness

Operationally, representativeness is achieved when (1) appraisal procedures used to value the sample parcels are similar to procedures used to value the corresponding population, (2) sample properties are not unduly concentrated in certain areas or types of property whose appraisal levels differ from the general level of appraisal in the population, and (3) sales prices or independent appraisals provide good surrogates for market values.

The first requirement is generally met unless appraisers value sample parcels differently from nonsample parcels, or unless appraisals of sample parcels were done at a different time than appraisals of nonsample parcels (see section 10).

The second requirement relates to the extent to which appraisal performance for the sample reflects appraisal performance for the population. For example, assume that the stratum (population) being analyzed is all vacant land parcels in a county and that there are 2,400 parcels in the stratum, of which 800 are in an active suburban area. Assume also that there were 150 vacant land sales during the period of study. If 100 of them were from this area, a ratio study would be likely to either under- or overstate assessment accuracy, depending on whether vacant land in the area was appraised more or less accurately than other vacant land in the county. Note, however, that if the subject area were appraised at a similar level of appraisal as other areas, the results of the study would be accurate, even though the sales were not geographically representative of the population.

Lack of locational or physical representativeness does not necessarily mean that a ratio study is invalid. However, it does make the study suspect and suggests the need to verify that the under- and overrepresented properties are appraised at the same level. Sometimes additional stratification may be required. In the present example, vacant land could be stratified by area, so that sales from the rapidly growing suburban area would be used to evaluate appraisal performance only in that area. Weighting of strata can be used to compensate for over- or under-representation of particular types of property in a sample (see section 7.3.5).

It is good practice to compare profiles of the sample and population of properties based on such key characteristics as appraised value, location, age, and size. If the profiles are similar, barring selective reappraisal of sold properties, ratio studies will tend to be valid.

Another factor affecting the representativeness of samples is the number of sales or independent appraisals used in the study. In general, barring complications in meeting the requirements discussed above, larger samples tend to be more representative, provided the homogeneity of the populations is considered. That is, homogeneous strata (having less variance) require smaller samples than heterogeneous strata, although one must avoid creating samples that are too small for analysis. These relationships are addressed more fully in section 8.

5.5.2 High-Value Properties

Assessment jurisdictions often contain unique, high-value properties (for example, properties that constitute more than 5 percent of the value of a property class) that cannot reasonably be combined with other properties for purposes of the ratio study. For indirect equalization, inclusion of high-value parcels is necessary to attain representativeness. For instance, consider a population consisting of 1,000 properties, 999 of which range in value from $20,000 to $750,000, and one that is valued at $1 billion (a nuclear power plant). If the intended use of the ratio study is to estimate the general level and uniformity of appraisal in regard to the typical property, the sample need not include the $1 billion property. If the intended use of the ratio study is to estimate the total market value in the jurisdiction, however, exclusion of the power plant would seriously distort the study.

Because of their high value, these properties cannot be ignored or assumed to be appraised at the statutory or general level in indirect equalization studies. The equalization agency should conduct an appraisal of such properties (the appraisals may be trended for several years) or audit and adjust as necessary the values developed by the local jurisdiction.

6. Acquisition and Analysis of Sales Data

Most ratio studies use sales prices as indicators of market values. In addition, sales data are important in appraisal ratio studies and in any effective and efficient mass appraisal system. To serve such functions, sales information must first be acquired and screened. In many instances, it is also necessary to confirm sales or make adjustments to sales prices.

6.1 Information Required

The data needed to screen sales, make any necessary adjustments to sales prices, compute sales ratios, and update ownership information are listed below.

1. *Full consideration involved.* This is the total amount paid for the property, including the cash down payment and amounts financed. The sale price is the most essential item of information concerning the sale, and its accuracy must be carefully scrutinized. In many jurisdictions it is common practice in deeds of conveyance to state considerations in such terms

as "one dollar plus other due and just consideration." These amounts are rarely the actual selling price and should be ignored in favor of information from the buyer and seller or other reliable source. In other cases, particularly when large amounts of money or professional investors are involved, sales prices may be deliberately under- or overstated for various reasons.

2. *Names of buyer and seller.* This information permits the assessor to maintain a current record of the owners of all property in the jurisdiction. Transfer documents often refer to the buyer as the grantee or transferee and to the seller as the grantor or transferor.

3. *Addresses, phone numbers, and other contact information of buyer and seller or their legal designee.* This information helps to identify more positively the parties to the sale. If the buyer will not reside at the property, the buyer's address may be needed for future correspondence. If the seller has established a new address, this information will aid the assessor in contacting the seller regarding the sale.

4. *Relationship of buyer and seller.* It is important to know whether the buyer and seller are related individuals or corporate affiliates because such sales often do not reflect market value.

5. *Legal description, address, and parcel identifier.* If each parcel is assigned a unique parcel identifier and if this number is noted on the document at the time it is recorded, then the assessor can locate the parcel in the files directly. If not, the legal description or street address is essential to locate the parcel. The name of the seller alone may not suffice, because (a) the seller may own multiple properties or (b) the seller may not appear in the tax index at all (for example, when a mortgagee pays the taxes).

6. *Type of transfer.* It is crucial to identify whether or not a sale is an "arm's-length" transfer. See section 6.4.1 for examples of transfers that do not meet this requirement. The type of deed (for example, warranty deed, land contract, quit claim deed, sheriff's deed, and so forth) can reveal important information about the type of transfer. Therefore, if the sources of sales data do not include copies of deeds, the type of deed should be specifically required.

7. *Time on the market.* Sales that have not been exposed to the open market or that have been on the market an abnormally long time may not represent market value.

8. *Interest transferred.* It is crucial to identify whether or not the entire bundle of rights (fee simple) to the property has transferred. If it has not, the sale price cannot be considered representative of the total market value of the property. For example, in some transactions, only a life tenancy ("life estate") may be conveyed, or the seller may retain mineral or other rights to the property. Similarly, the sale price of a property encumbered by a lease may not reflect the total value of the property.

9. *Type of financing.* In analyzing the sale, it is helpful to know the amount of down payment; the type, remaining amount, and interest rates of notes secured by mortgages or deeds of trust assumed by the buyer; and the value of any stocks, bonds, notes, or other property passed to the seller. It is also important to know whether the sale conveys title to the property or is a land contract, in which title is not conveyed until some time in the future, typically several years.

10. *Personal property.* A sales ratio study requires knowledge of the amount paid for the real property. The sale document ideally would note the type and value of any significant personal property items included in the transaction. Common items that normally constitute little value, such as carpets, draperies, free-standing appliances, and other interior furnishings can be ignored. (See section 6.4.4.)

11. *Date of transfer.* This is the date on which the sale was closed or completed. The date the deed or other transfer document was recorded can be used as a surrogate, provided there was no undue delay in the recording. If recording delay is undue, the date of the deed or transfer instrument should be used.

12. *Instrument number.* This number indicates where the deed is located in the official records and thus can be important in researching sales or leases and identifying duplication.

Sales data files should reflect the physical characteristics of property when sold. If significant physical changes have been made to a property between the sale date and the assessment date, the sale should not be used for ratio studies. (The sale may still be valid for mass appraisal modeling by matching the sale price against the characteristics that existed on the date of sale.)

6.2 Sources of Sales Data

The best sources of sales data are copies of deeds or real estate transfer affidavits containing the full consideration and other particulars of the sale (see section 6.1). Assessing officers in jurisdictions without laws mandating full disclosure of sales data to assessing officials work under a severe handicap and should seek legislation that provides for such disclosure.

1. *Real estate transfer documents.* These documents include (1) copies of deeds and land contracts, (2) copies of real estate transfer affidavits, and (3) closing statements.

2. *Buyers and sellers.* Buyers and sellers of real property can be contacted directly to secure or confirm sales data. Means of contact include sales questionnaires (see appendix), telephone interviews, and personal interviews.

3. *Third-party sources.* Third-party sources include multiple listing agencies, real estate brokers and agencies, government and private fee appraisers, attorneys, appraisal organizations, and others. Of particular value are those individuals or agencies that publish lists of sales or provide sales on a computer-readable medium.

6.3 Confirming Sales

6.3.1 Importance of Confirmation

The usefulness of sales data is directly related to the completeness and accuracy of the data. Sales data should be routinely confirmed or verified by contacting buyers, sellers, or other knowledgeable participants in the transaction. In general, the fewer the sales in a stratum, the less common or more complex the type of property, and the more atypical the sale price, the greater the effort should be to confirm the particulars of the sale. It may be sufficient to confirm single-family sales by audit or exception.

Property tax supervisory agencies whose primary source of sales data is local assessors should routinely conduct a thorough audit to confirm that sales processing and validation procedures are being followed uniformly.

6.3.2 Methods of Confirmation

In general, the completeness and accuracy of sales data are best confirmed by requesting the particulars of a sale from parties to the sale. A sales questionnaire, which requests the type of information listed in section 6.1, is one practical means of confirming sales. Interview forms, with space to record the same types of information, also can be used for telephone and personal interviews. The appendix contains a model sale confirmation questionnaire (additional sample sales questionnaire and interview forms can be found in *Improving Real Property Assessment* [IAAO 1978, 95–104]).

Mailed sales questionnaires should be as concise as possible and should include a postage-paid return envelope. This makes the form easier to complete and increases the likelihood that it will be returned. Official stationery should be used, and requests for information should bear an authorized signature. The request should state the purpose of the questionnaire, cite any specific statutory authority, and indicate how quickly the questionnaire should be returned. The request should also include the telephone number and, if possible, the name(s) of the person(s) to contact should the respondent have any questions.

6.4 Screening Sales

Once stratification has occurred and sample requirements have been determined (see sections 4.4 and 5.5), sales used in a ratio study must be screened to ensure that they reflect the market value of the real property transferred. Specific objectives of sales screening are to ensure the following:

1. Sales used in ratio studies reflect to the maximum extent possible the conditions contained in the definition of market value.

2. Sales prices reflect only the market value of the real property transferred and not the value of personal property, financing, leases, or other parcels of real property.

3. Only sales that occurred during the period of analysis are used.

4. Sales are excluded from the ratio study only with good cause (for example, when they compromise the reliability of the study).

Every arm's-length, open-market sale that appears to meet the conditions of a market value transaction should be included in the ratio study unless (a) data for the sale are incomplete, unverifiable, or suspect, (b) the sale fails to pass one or more specific tests of acceptability, or (c) a representative sample of sales that occurred during the study period can be randomly selected to provide reliable statistical measures.

The sales analyst should take the position that all sales are candidates for the ratio study unless sufficient and compelling information can be documented to show otherwise. If sales are excluded without substantiation, the study may appear to be subjective. Codes can be established for each condition that leads to a sale being rejected or questioned, so that the analyst can include or exclude sales with a given code.

No single set of sales screening rules or recommendations can be universally applicable for all uses of sales data or under all conditions. Analysts must use their judgment, but should not be arbitrary. To help analysts make wise and uniform judgments, screening procedures should be in writing, and each sales analyst should be thoroughly familiar with these procedures as well as with underlying real estate principles.

6.4.1 Sales Generally Invalid for Ratio Studies

The following sales are often found to be invalid for ratio studies and can be automatically excluded, unless a larger sample size is needed and further research is conducted to determine that sales are open-market transactions.

1. *Sales involving government agencies and public utilities.* Such sales may involve an element of compul-

sion and often occur at prices higher than would otherwise be expected. On the other hand, sales by governmental agencies of surplus property or of redevelopment sites tend to be at favorable prices.

2. *Sales involving charitable, religious, or educational institutions.* A sale to such an organization may involve an element of philanthropy, and a sale by such an organization may involve a nominal consideration or restrictive covenants.

3. *Sales involving financial institutions.* A sale in which the lienholder is the buyer may be in lieu of a foreclosure or a judgment and the sale price may equal the loan balance only.

   Sales in which a financial institution is the seller should be viewed cautiously. Such sales tend to occur at lower prices than comparable, conventionally financed sales. If the lienholder can confirm that the asking price was based on a market-value appraisal or if such sales constitute a major portion of the market (as in depressed areas), these sales can be used, provided they are made on the open market and satisfy the other conditions of a market value transaction. In such cases, adjustments should be considered for any differences in price from conventionally financed sales.

4. *Sales between relatives or corporate affiliates.* Sales between relatives are usually non-open-market transactions and tend to occur at prices lower than would otherwise be expected. Sales between corporate affiliates may be made only to obtain financing or to adjust corporate accounts. Knowledge of corporate relationships is usually required to identify corporate affiliates, although a buyer and seller at the same address may indicate an affiliation.

5. *Sales of convenience.* Such sales are intended to correct defects in a title, create a joint or common tenancy, or serve some similar purpose. In such situations, the sale price is usually nominal. Sales of convenience can be identified by deed type, the statement of the interest transferred, or the relationship of the buyer and seller.

6. *Sales settling an estate.* A conveyance by an executor or trustee under powers granted in a will may not represent fair market value, particularly if the sale takes place soon after the will has been filed and admitted to probate in order to satisfy the decedent's debts or the wishes of an heir. If, however, the sale is not forced and meets the other conditions of market value, estate sales can be regarded as valid.

7. *Forced sales.* Such sales include those resulting from a judicial order. The seller in such cases is usually a sheriff, receiver, or other court officer. If the property sells at an auction that is well-advertised and well-attended, the sale price may be taken as a valid indication of market value. However, auctions in which the receiver is required to accept whatever bid is offered are known as *absolute auctions* and produce sales that are always invalid for consideration in ratio studies.

8. *Sales of doubtful title.* Sales in which title is in doubt tend to be below market value. When a sale is made on other than a warranty deed, there is a question of whether the title is merchantable. Quit claim deeds and trustees' deeds are examples. Of course, a warranty deed may not be given simply because the seller wishes to avoid personal liability of any sort. Quit claim deeds are frequently used in commercial sales and in sales involving large amounts of money.

6.4.2 Sales with Special Conditions

Sales with special conditions may be open-market sales but must be verified thoroughly and used with caution in ratio studies.

1. *Trades.* In a trade, the buyer gives the seller one or more items of real or personal property as all or part of the full consideration. If the sale is a pure trade with the seller receiving no money or securities, the sale should be excluded from analysis. If the sale involves both money and traded property, it may be possible to include the sale in the analysis if the value of the traded property is stipulated, can be estimated with accuracy, or is small in comparison to the total consideration. However, transactions involving trades should be excluded from the analysis whenever possible, particularly when the value of the traded property appears substantial.

2. *Partial interests.* A sale involving the conveyance of less than the full interest in a property should be excluded from the analysis unless several sales of partial interests in a single property take place at the same time and provided that the sum of the partial interests equals the fee-simple interest. Then the sum of the sales prices of the partial interests can sometimes be assumed to indicate the sale price of the total property. At other times, however, the purchase of such partial interests is analogous to an assemblage in which a premium may have been paid.

3. *Land contracts.* Land contracts and other installment purchase arrangements in which title is not transferred until the contract is fulfilled require careful analysis. Deeds in fulfillment of a land contract often reflect market conditions several years in the past, and such dated information should be excluded from analysis. Sales data from land contracts may also reflect the value of the financing arrangements. In such instances, if the transaction is recent, the sale price should be adjusted for financing (see section 6.5.2).

4. *Incomplete or unbuilt common property.* Sales of condominium units and of units in planned unit developments or vacation resorts often include an interest in common elements (for example, golf courses, clubhouses, or swimming pools) that may not exist or be usable on the date of sale or on the assessment date. Sales of such properties should be examined to determine whether prices might be influenced by promises to add or complete common elements at some later date. Sales whose prices are influenced by such promises should either be excluded from the analysis or the sales prices should be adjusted to reflect only the value of the improvements or amenities in existence on the assessment date.

5. *Auctions.* Generally, auction sales of real property tend to be at the lower end of the price spectrum. Auction sales that have been well advertised and well attended may be valid for consideration in ratio studies. The seller must also have the option to set a minimum bid on the property or the right of refusal on all bids (*with reserve*) in order for the sale to be considered valid.

6.4.3 Commercial Transactions by Large Business Entities

Acquisitions or divestments by large corporate entities, pension funds, or real estate investment trusts (REITs) may be dictated by strategic business interest and can involve a large number of parcels, multinational properties, and package deals. Package deals are sales of multiple properties packaged and sold as a unit. Often, the sale price for a package deal cannot be accurately allocated to the individual parcel of real estate in the relevant taxing jurisdiction and, if this is the case, the resulting ratio may not be accurate. Large commercial transactions may also include deed restrictions, restrictive covenants, or special circumstances (such as favorable income tax treatment) that can influence market value. Sales involving large commercial entities must be investigated thoroughly to determine that sales prices fairly reflect local market conditions and the supply and demand factors relating to the specific property type and location.

6.4.4 Sales Involving Personal Property

Sales screening includes determining the value of any significant personal property included in the sale. Personal property that amounts to more than 5 percent of the total sale price should be considered significant. Personal property often included in sales prices includes such tangibles as machinery, furniture, and inventories and such intangibles as franchises, licenses, and agreements-not-to-compete. Ordinarily, however, it is not necessary to consider goodwill, going-concern value, business enterprise value, or the like, unless the value of these intangible assets has been itemized in a sales contract or a formal appraisal has been prepared by either party.

It is necessary to decide whether each item included in the sale should be classified as real or personal property. (See *Standard on Valuation of Personal Property* [IAAO 1996], which provides guidance on classification of property as real or personal.)

6.4.5 Sales of Exempt Property

Sales of property not subject to taxation should be excluded from analysis.

6.5 Adjustments to Sales Prices

Sales prices used in ratio studies may need to be adjusted for financing, assumed leases, personal property, and date of sale. This is especially true for nonresidential properties. The real property tax is based on the market value of real property alone as of a specific date. This value may not be the same as investment value (that is, the monetary value of a property to a particular investor) and does not include the value of personal property or financing arrangements. Thus, failure to make required adjustments for these factors gives unsatisfactory estimates of market value for the real property.

If adjustments for more than one purpose are to be made, they should be made in the following order:

1. Adjustments that develop or isolate the price paid for taxable real property. These include adjustments for personal property received by the buyer, property taken in trade by the seller, the combination of partial interest sales, and incomplete or unbuilt common property.

2. Adjustments that convert the price to a better representation of the market value of the property on the date of sale. These include adjustments for financing and assumed leases.

3. Adjustments for differences in market value levels between the date of sale and the date of analysis.

Procedures for adjusting sales prices should be documented, and the adjustment factors supported by market data. These requirements imply an ongoing study of local real estate prices, interest rates, and financing practices. Unsubstantiated adjustments will jeopardize the acceptance accorded a ratio study by making it appear subjective.

6.5.1 Adjustments for Personal Property

Sales prices should be adjusted by subtracting the value of personal property received by the buyer. Such adjustments should be based on evidence of the effect of personal property on the sale price. In practice, the effect cannot be proven. Adjustments must be based on the best available evidence. Ordinary window treatments, outdated models of free-standing appliances, and common-grade used furniture do not usually influence the sale price of real

property and do not require adjustment. If the dollar amount of the personal property provided by a party to the sale appears reasonable and consistent with market prices or if the buyer and seller are in agreement as to the effect of personal property on the sale price, that estimate can be used for the adjustment. For personal property items with higher value, a reliable alternative method is to appraise the personal property and subtract its value from the sale price. Where these options are not practical and the value of the personal property appears typical, a standard adjustment based on a study of similar properties can be made. Standard adjustments should be determined from reasonably current market analysis. Where market analyses show considerable variation in the percent of personal property included in sales, standard adjustments should not be used. If a portion of the sale price is designated as being for personal property and if that amount appears reasonable and can be supported by other market data, it should be deducted from the sale price.

When evidence of the value of personal property is not provided, no adjustment is necessary. If the stated value of personal property is more than 5 percent of the total sale price for residential property or more than 25 percent for commercial property, the sale should be excluded unless the sales sample is small and there is strong evidence to support the value estimate of the personal property.

6.5.2 Adjustments for Financing

When financing reflects prevailing market practices and interest rates, sales prices require no adjustment for financing. Adjustments should be considered when

1. the seller and lender are the same party and financing is not at prevailing market rates,

2. the buyer assumes an existing mortgage at a nonmarket interest rate, or

3. lenders charge the seller "points" (a percentage of the loan amount) for making money available to the purchaser/borrower. Points paid by the borrower are part of the down payment and do not require adjustment.

As with personal property, the preferred means of adjusting for financing is by individual parcel. In the first two instances above, downward adjustments are warranted when (1) the loan appears to be well secured and the contract interest rate is less than the market interest rate, or (2) the loan appears not to be well secured and the contract interest rate is lower than that required by the market for a loan of equal risk. The amount of adjustment can be computed by capitalizing the difference between monthly payments based on the required market interest rate and those based on the actual interest rate. Particularly for residential property, however, the sale price often does not reflect this full amount because of the uncertainty of the holding period, tax considerations, and the like. Market analysis using paired sales (sales of similar properties, some with and some without conventional financing) or statistical techniques can correct for such factors. When the seller pays points, the sale price should be adjusted downward by the value of the points.

Adjustments for financing require data on actual and market interest rates, the amount of the loan, and the term and amortization provisions of the loan. Obtaining and properly analyzing such data, as well as determining the extent to which the market actually capitalizes nonmarket financing, is difficult, time-consuming, and requires skilled sales analysts. Whether such adjustments should be made depends in part on available sample size, completeness of the available information, staff expertise, and the likely effect of such adjustments on the ratio study. As with personal property adjustments, when required data are unknown or incomplete, it may be possible to compute typical adjustments based on a study of similar properties. For example, if it is known that the seller and lender in a vacant land sale are the same party, but data on terms of the financing are not available, a typical adjustment could be made based on typical interest rates observed in other seller-financed vacant land sales. Although this approach is inferior to a full analysis of the sale based on complete information, it is preferred to an arbitrary adjustment and, if the sample size is small, may be preferred to excluding the sale.

6.5.3 Adjustments for Assumed Leases

The sale price of a property encumbered by a long-term lease should be adjusted by the difference between the present worth of the two income streams if the contract rent differs significantly from market rent. If the contract rent exceeds market rent, the present worth of the difference in the two income streams should be *subtracted* from the sale price. If the contract rent is less than current market rent, the present worth of the difference in the two income streams should be *added* to the sale price.

6.5.4 Adjustments for Time

There should be a program of monitoring changes in price levels over time and adjusting sales prices for time as required. Such a program is an important component of a mass appraisal system as well as of a ratio study. Time adjustments must be based on market analysis and supported with appropriate documentation.

Valid time-adjustment techniques include (1) tracking sales and appraisal ratios over time; (2) including date of sale as a variable in regression or feedback models; (3) analyzing resales; (4) comparing per-unit values over time in homogeneous strata, such as a subdivision or condominium complex; and (5) isolating the effect of time through paired sales analysis. These techniques are discussed in Gloudemans (1990; 1999), IAAO (1990, appendix 5-3), and IAAO (1978, section 4.6).

It is particularly important to monitor changes in price levels over time in ratio studies made for equalization purposes where the objective of the study is to estimate the level of appraisal on a specific assessment date. If sales prices have generally been rising, ratios for sales that occurred after the assessment date will tend to understate the overall level of appraisal. Similarly sales ratios for sales that occurred before the assessment date will tend to overstate the level of appraisal. If prices are generally declining, an opposite pattern will result.

To the extent that sales data permit, changes in price levels should be monitored and time adjustments made by area and type of property, because different segments of the property base tend to change in value at different rates.

6.5.5 Other Adjustments

Adjustments to sales prices should not be made for real estate sales and brokerage commissions; closing costs such as attorney's fees, transfer taxes, and title insurance; and current or delinquent property taxes. Exceptions to this general rule occur when the buyer agrees to pay real estate commissions and delinquent property taxes, in which case the amounts of the payment should be added to the sale price. Other exceptions occur when the seller agrees to pay expenses normally paid by the buyer. Such expenses include loan origination fees and repair allowances. Such payments by the seller should be deducted from the sale price.

6.5.6 Special Assessments

Special assessments are used to finance capital improvements or provide services adjacent to the properties they directly benefit. Typically, the property owner is obligated to make annual payments of principal and interest to a local unit of government over a specified number of years. The sale price of a property encumbered by such a special assessment may require adjustment if the current balance of the defrayed amount is significant. The sale price can be adjusted upward to account for this lien. As with other financing adjustments, the collection and analysis of data can be time-consuming, and it may be difficult to determine the extent to which the market actually capitalizes this assessment (see section 6.5.2). If the effect on market value can be ascertained, an adjustment should be made.

6.5.7 Adjustments for Statutorily Imposed Value Constraints

Most states and provinces require appraisal of certain classes of property using statutorily prescribed methods of appraisal that are intended to produce constrained value that is less than market value. The most common class of property to which such constraints apply is farmland and rangeland that qualifies for agricultural use value. However, constraints may apply to subsidized housing, mineral land, and other classes.

When the purpose of the ratio study is direct or indirect equalization, sales prices must be adjusted as if the selling parcel were subject to the same constraints. If this cannot be done, independent appraisals, which employ the required constraints, should be used to determine the level of appraisal in a manner that is consistent with the statutory constraints. For example, assume that statutory restrictions require a fixed or artificially high capitalization rate to be used in determining farmland value. If unadjusted farmland sales were to be used, the resulting ratios would be low and could lead to improper equalization decisions. Instead, independent appraisals using the required capitalization rate should be done. These would lead to ratios that would correctly allow for the statutory constraint.

Use of constrained values produces ratio study results that do not provide information on the true level of appraisal in relation to market value. Use of constrained values is appropriate for equalization. However, when the purpose of the ratio study is to determine the overall quality of assessments or the amount of benefit being awarded by a given statutory constraint on appraised value, the unadjusted sale price or independent market value appraisal must be used. Often, procedural audits can be used as adjuncts to more traditional ratio studies. These can be particularly effective when the purpose is to judge overall appraisal quality and precise, quantitative statistical measures are not required.

6.6 Outlier Ratios

Outlier ratios are very low or high ratios as compared with other ratios in the sample. When the sample is small, outlier ratios may distort calculated ratio study statistics. Some statistical measures, such as the median ratio, are resistant to the influence of outliers. However, the COD and mean are sensitive to extreme ratios.

Outlier ratios can result from any of the following:

1. an erroneous sale price
2. a nonmarket sale
3. unusual market variability
4. a mismatch between the property sold and the property appraised
5. an error in the supplemental appraisal performed by the oversight agency
6. an error in the assessing unit's appraisal of an individual parcel
7. an error in the assessing unit's appraisals of a subgroup

The preferred method of handling an outlier ratio is to subject it to additional scrutiny to determine whether the sale is a nonmarket transaction, a correctable error exists, or the property reduces the representativeness of the sample (see section 5.5 on representative samples). If a sale is found to be nonmarket, it should be excluded. If

Table 1
A Distribution-free Method for Locating Outliers and Extreme Outliers

The following procedure will identify outlier ratios that fall more than 1.5 times beyond the range of the middle 50 percent of the arrayed sample. Ratios that exceed 3 times this range can be labeled as extreme outliers.

Locating trim boundaries

Data set before trimming

Rank	Ratio (A/S)
1	0.611
2	0.756
3	0.762
4	0.853
5	0.867
6	0.909
7	0.925
8	0.944
9	1.014
10	1.052
11	1.178
12	1.367
13	1.850
14	2.500
Median ratio	0.935
COD	32.271

Steps to locate trim boundaries

1. Locate the first quartile point
Formula to locate the first quartile:
(0.25 × number of ratios) + 0.25
(0.25 × 14 ratios) + 0.25 = 3.75
3.75 is three-quarters between the third and fourth ranked ratios.
Ratio 3 = 0.762 Ratio 4 = 0.853
Three-quarters between = (0.853 – 0.762) × 0.75 = 0.068
The first quartile point = 0.762 + 0.068 = 0.830

2. Locate the third quartile point
Formula to locate the third quartile
(0.75 × number of ratios) + 0.75
(0.75 × 14 ratios) + 0.75 = 11.25
11.25 is one-quarter between the eleventh and twelfth ranked ratios.
Ratio 11 = 1.178
Ratio 12 = 1.367
One-quarter between = (1.367 – 1.178) × 0.25 = 0.047
The third quartile point = 1.178 + 0.047 = 1.225

3. Compute the interquartile range
The distance between the first and third quartile = interquartile range
1.225 – 0.830 = 0.395

4. Establish the lower boundary
The lower trim point = first quartile – (interquartile range × 1.5 or 3.0)
0.830 – (0.395 × 1.5) = 0.238 0.830 – (0.395 × 3.0) = – 0.355

5. Establish the upper boundary
The upper trim point = (interquartile range × 1.5 or 3.0) + third quartile
(0.395 × 1.5) + 1.225 = 1.818
(0.395 × 3.0) + 1.225 = 2.410

Outliers eliminated after 1.5× trimming		Extreme outliers eliminated after 3.0× trimming	
Rank	Ratio (A/S)	Rank	Ratio (A/S)
1	0.611	1	0.611
2	0.756	2	0.756
3	0.762	3	0.762
4	0.853	4	0.853
5	0.867	5	0.867
6	0.909	6	0.909
7	0.925	7	0.925
8	0.944	8	0.944
9	1.014	9	1.014
10	1.052	10	1.052
11	1.178	11	1.178
12	1.367	12	1.367
		13	1.850
Median ratio	0.917	Median ratio	0.925
COD	15.649	COD	22.012

an error exists that can be corrected (for example, a data entry error or an error in a supplemental appraisal), the error should be corrected and the property left in the sample.

One extreme outlier can have controlling influence over some statistical measures. Particular care must be taken to identify outliers if point estimates are used to make inferences about population level or uniformity. If, after proper verification, screening, and editing, an outlier with a nonrepresentative ratio remains in a study, statistical results will not reflect population level and uniformity. The potential distortion is greater when sample size is small. If outliers can be identified, trimming procedures are acceptable methods for creating a more representative sample. One outlier identification method is based on the interquartile range (see table 1). However, because of the skewed distribution of ratios this procedure may locate only extremely high ratios. If one or two high outlier ratios are trimmed from a small sample, the statistical measures of level may be shifted significantly lower. (See Tomberlin [1997] and Hoaglin, Mosteller, and Tukey [1983] on trimming small samples.)

Detected outliers should be reported and may be treated in a variety of ways, including trimming (see D'Agostino

and Stephens [1986]). If outliers are to be considered for removal, the analyst may select a procedure to trim all or just the extreme outliers (see table 1). If a trimming method has been used to reject ratios from the sample, this fact must be stated. Arbitrary trim points, for example, trimming of all ratios less than 50 percent or greater than 150 percent, tend to distort results and should not be employed. Outlier trimming is not mandatory; however, sales with extreme ratios must be thoroughly validated and determined to be highly trustworthy observations because they may play a pivotal role in the ratio study outcome.

Outlier ratios can seriously distort a statistical analysis. However, if outlier ratios tend to be concentrated in certain areas or other subsets of the sample, they may point directly to systematic errors in the appraisal process and should be retained if they are truly representative. In appraisal ratio studies, a property should be excluded only if it no longer exists, has been significantly changed, or can no longer be reliably appraised for equalization purposes.

Ratio study reports or accompanying documentation should clearly state the basis for excluding outlier ratios (see section 12.1). Statistics calculated from trimmed distributions, obviously, cannot be compared to those from untrimmed distributions or interpreted in the same way. This is especially problematic when making interjurisdictional comparisons. For this reason, oversight agencies may wish to promulgate uniform trimming procedures.

7. Ratio Study Statistics and Analyses

7.1 Introduction

Once data have been properly collected, reviewed, and assembled, statistical analysis can begin. This involves several steps. First, a ratio should be calculated for each observation in the sample by dividing the appraised (or assessed) value by the sale price or other independent estimate of market value. Data on individual ratios are required for every ratio study statistic or test except those relying solely on weighted mean ratios. Second, graphs and exhibits can be developed that show the distribution of the ratios (see section 7.2). Third, exploratory data analysis, including tests of the hypotheses of normality should be conducted (see section 7.7). Fourth, ratio study statistics of both appraisal level and uniformity should be calculated. In addition, reliability measures should be calculated for appraisal level statistics. Reliability measures (hypothesis tests or confidence intervals) for uniformity statistics are also possible (sections 7.3 through 7.7). Finally, appropriate hypothesis tests for horizontal and vertical equity or other hypotheses being studied should be conducted. (Table 2 gives a data set and some partial computations of statistical measures. Statistical measures and their computations are discussed in detail beginning in section 7.3.1).

7.2 Data Displays

Displays or exhibits that provide a profile or picture of ratio study data are useful for illustrating general patterns and trends, particularly to nonstatisticians. The particular form of the displays, as well as the data used (for example, sales prices, sales ratios, and property characteristics) will depend on the purposes of the particular display. Types of displays useful in ratio studies include arrays, frequency distributions, histograms, line charts, scatter diagrams, schematic plots, and maps. The purposes of these displays include (1) indicating whether a sample is representative of the properties in a stratum, (2) indicating the degree of nonnormality in the distribution of ratios, (3) depicting the overall level of appraisal, (4) depicting the degree of uniformity, (5) comparing the level of appraisal or degree of uniformity among strata, and (6) detecting outlier ratios.

7.3 Measures of Appraisal Level

Estimates of appraisal level are based on measures of central tendency. They should be calculated for each stratum and for such aggregations of strata as may be appropriate. Several measures of central tendency (appraisal level) can be calculated in ratio studies. These include

- the median ratio
- the mean ratio
- the weighted mean ratio
- the geometric mean ratio

As many of these as are applicable in a given situation should be calculated.

When these measures are calculated on the data in a sample, the result is a point estimate, which is accurate for the sample but is only an indicator of the level of appraisal in the population. Confidence intervals around the measures of level provide indicators of the reliability of the sample statistics as predictors of the overall level of appraisal of the population (see section 7.5). It is important to note that compliance with appraisal level standards (see section 14) cannot be determined without the use of confidence intervals or hypothesis tests.

7.3.1 Median

The median ratio is the middle ratio when the ratios are arrayed in order of magnitude. If there is an even number of ratios, the median is the average of the two middle ratios. The median for the thirty-six ratios in table 2 is:

$$\frac{0.848 + 0.880}{2} = 0.864.$$

The median always divides the data into two equal parts and is less affected by extreme ratios than the other measures of central tendency. Because of these proper-

Table 2:
Example of Ratio Study Statistical Analysis

Data analyzed

Rank of ratio of observation	Appraised value (AV in $)	Market value (MV in $)	Ratio (AV/MV)
1	48,000	138,000	0.348
2	28,800	59,250	0.486
3	78,400	157,500	0.498
4	39,840	74,400	0.535
5	68,160	114,900	0.593
6	94,400	159,000	0.594
7	67,200	111,900	0.601
8	56,960	93,000	0.612
9	87,200	138,720	0.629
10	38,240	59,700	0.641
11	96,320	146,400	0.658
12	67,680	99,000	0.684
13	32,960	47,400	0.695
14	50,560	70,500	0.717
15	61,360	78,000	0.787
16	47,360	60,000	0.789
17	58,080	69,000	0.842
18	47,040	55,500	0.848
19	136,000	154,500	0.880
20	103,200	109,500	0.942
21	59,040	60,000	0.984
22	168,000	168,000	1.000
23	128,000	124,500	1.028
24	132,000	127,500	1.035
25	160,000	150,000	1.067
26	160,000	141,000	1.135
27	200,000	171,900	1.163
28	184,000	157,500	1.168
29	160,000	129,600	1.235
30	157,200	126,000	1.248
31	99,200	77,700	1.277
32	200,000	153,000	1.307
33	64,000	48,750	1.313
34	192,000	144,000	1.333
35	190,400	141,000	1.350
36	65,440	48,000	1.363

Note: Due to rounding, totals may not add to match those on following table, which reports results of statistical analysis of above data.

Results of statistical analysis

Statistic	Result calculated on preceding data
Number of observations in sample	36
Total appraised value	$3,627,040
Total market value	$3,964,620
Average appraised value	$100,751
Average market value	$110,128
Mean ratio	0.900
Median ratio	0.864
Geometric mean ratio	0.849
Weighted mean ratio	0.915
Price-related differential (PRD)	0.98
Coefficient of dispersion (COD)	29.8%
Standard deviation	0.297
Coefficient of variation (COV)	33.0%
Probability that population mean ratio is between 90% and 110%	49.7%
95% mean two-tailed confidence interval	0.799–1.000
95% median two-tailed confidence interval	0.684–1.067
95% weighted mean two-tailed confidence interval	0.806–1.024
Shape of distribution of ratios	Normal (based on binomial distribution)
Date of analysis	9/99/9999
Category or class being analyzed	Residential

ties, the median is the generally preferred measure of central tendency for direct equalization, monitoring appraisal performance, determining reappraisal priorities, or evaluating the need for a reappraisal.

7.3.2 Mean

The mean ratio is the arithmetic average of the ratios. It is calculated by summing the ratios and dividing by the number of ratios. In a normal distribution the mean will equal the median. In a distribution skewed to the right (typical of ratio study data), the mean will be greater than the median. Conversely, in a distribution skewed to the left, the mean will be less than the median. The mean of the data in table 2 is:

$$\frac{32.385}{36} = 0.900.$$

The mean is affected more by extreme ratios than the median. Also, for statistical reasons, the sample mean has a slight upward bias with respect to the true level of assessment, that is, it tends to overestimate slightly the true level of assessment (see IAAO [1990, chapter 20] and Gloudemans [1999, chapter 5] for a more detailed discussion). Regardless of the distribution of the data, the sample mean can be a valid estimator of the mean level of appraisal in the population, provided outliers are trimmed as appropriate. However the mean should not be used for indirect equalization if there are measurable differences in appraisal level of high- and low-value properties. In tests of a population of properties with a known mean and median, repeated samples from that population show that the median is closer to the population median than the mean is to the population mean. In data commonly containing outliers, the trimmed mean can be substituted for the mean (Gloudemans 1999, chapter 3; see section 6.6 for trimming procedures).

7.3.3 Weighted Mean

The weighted mean ratio is the weighted average of the ratios where the weights are proportionate to the sales prices. That is, the weighted mean gives weight to each dollar of value in the sample, whereas the median and mean give equal weight to each parcel. The weighted mean is an important statistic in its own right and also is used in computing the price-related differential (PRD), a measure of uniformity between high- and low-value properties (see section 7.6).

The weighted mean can be calculated by (1) summing the appraised values, (2) summing the sales prices, and (3) dividing the first result by the second. The weighted mean is also called the "sum of the aggregates" or the "aggregate ratio." In table 2 the weighted mean is:

$$\frac{\$3,627,040}{\$3,964,620} = 0.915.$$

Because of its dollar weighting feature, the weighted mean is most appropriately used in indirect equalization, where one seeks to estimate the total dollar value of the jurisdiction. When relying on the measure, however, outliers should be carefully reviewed (and deleted if appropriate), since they can strongly affect the weighted mean, particularly when they occur for high-value properties.

7.3.4 Geometric Mean

The geometric mean ratio is calculated by multiplying the ratios in the sample together and finding the nth root of the result, where n equals the number of ratios. For the data in table 2 the geometric mean is:

$$(0.348 \times 0.486 \times \ldots \times 1.363)^{1/36} = 0.849$$

The geometric mean is less influenced by extreme ratios than the mean and weighted mean. Unless every ratio in the sample is identical, the geometric mean will be less than the mean. If the ratios vary widely, it may lie considerably below the other measures of central tendency. The geometric mean is not appropriate for equalization but can be useful as a way of flagging the presence of outliers and in time-series analysis. Large differences between the geometric mean and the mean tend to indicate the presence of significant outlier ratios.

7.3.5 Determining the Overall Ratio for Combined Strata

Measures of central tendency of aggregations of strata (for example, an overall measure of central tendency for a jurisdiction based on the measures of central tendency of the various strata) should be calculated by weighting the measures of central tendency of the respective strata. For purposes of monitoring appraisal performance, the most straightforward and generally preferred approach is to weight the median ratio of each stratum on the basis of the relative number of properties in the stratum. In other words, the weighting factor for a stratum would be the total number of properties in the stratum divided by the total number of properties in the jurisdiction or class.

For indirect equalization or the evaluation of a discrimination claim, the weight assigned a measure of central tendency of a stratum should be proportional to the share of that stratum's total estimated market value. Because number of parcels bears only a loose relationship to dollar value, weighting by number of parcels is not appropriate for indirect equalization.

A method of calculating such an overall ratio is to (1) divide the total appraised (or assessed) value of each stratum by the appropriate measure of central tendency to obtain an estimate of the total market value of taxable property in the stratum; (2) sum the estimates of total market value to obtain an estimate of the total market value of taxable property in the jurisdiction or class of property; and (3) divide the total appraised (or assessed) value of the jurisdiction or class of property by the

estimated total market value. Table 3 contains a simplified example.

For direct equalization, one can weight measures of central tendency (generally the median) based on either number of parcels or dollars of estimated market value, depending on whether one wants to equalize on a parcel- or dollar-weighted basis.

7.3.6 Contrasting Measures of Appraisal Level

Because it gives equal weight to each ratio and is unaffected by extreme ratios, the median is the preferred measure of central tendency for monitoring appraisal performance and direct equalization. Although the mean ratio is also a parcel-based measure, it can be greatly affected by extreme ratios and can only be relied upon if the sample is of adequate size and contains few outliers.

For indirect equalization, the preferred measure is the weighted mean (the measure used in table 3), which gives equal weight to each dollar. However, when samples are small, exhibit high dispersion, or contain outliers, the median, a transformed weighted mean, or other robust estimator can be substituted.

Table 4 summarizes the preferred measures of central tendency for the three broad purposes of indirect equalization, direct equalization, and the general monitoring of appraisal performance.

7.4 Measures of Variability

Measures of dispersion or variability relate to the uniformity of the ratios and should be calculated for each stratum in the study. In general, the smaller the measure, the better the uniformity, although extremely low measures can signal a flawed study, nonrepresentative appraisals, extremely homogeneous properties or stable markets. Note that as market activity changes or as the complexity of properties increase, the measures of variability usually increase, even though appraisal procedures may be equally valid.

7.4.1 Coefficient of Dispersion

The most generally useful measure of variability is the coefficient of dispersion (COD). The COD measures the average percentage deviation of the ratios from the median ratio and is calculated by (1) subtracting the median from each ratio, (2) taking the absolute value of the calculated differences, (3) summing the absolute differences, (4) dividing by the number of ratios to obtain the "average absolute deviation," (5) dividing by the median, and (6) multiplying by 100. For the data in table 2,

$$\text{average absolute deviation} = 9.271 \div 36 = 0.2575;$$

$$\text{COD} = (0.2575 \div 0.864) \times 100 = 29.8.$$

Table 3
Illustration of Combining Measures of Central Tendency

Data for properties in the study

Stratum (1)	Total assessed value (2)	Total sale price (3)	Weighted mean (2)÷(3) (4)	Total assessed value of stratum (5)	Indicated market value of stratum (6)
Residential	$3,000,000	$4,000,000	0.750	$600,000,000	$ 800,000,000
All other	950,000	1,000,000	0.950	400,000,000	421,000,000
Total				$1,000,000,000	$1,221,000,000

Overall ratio = $1,000,000,000/$1,221,000,000 = 0.819

Table 4
Preferred Measure of Central Tendency (Appraisal Level) by Purpose of Ratio Study

	Indirect equalization	Direct equalization	Monitoring performance
Median	*	X	X
Mean	**	**	**
Weighted mean	X***	—	—
Geometric mean ratio	—	—	—

* Medians calculated for each stratum should be weighted by estimated market value.
** May be acceptable under certain circumstances (see discussion above); if used for indirect equalization, measures for each stratum should be weighted by estimated market value (see Sherrill and Whorton 1991).
*** Caution should be exercised when the sample contains outliers or properties with unusually high values.

The COD has the desirable feature that its interpretation does *not* depend on the assumption that the ratios are normally distributed. Standards for interpreting CODs are contained in section 14.2. Note that the COD represents the mean, not the median, percent deviation from the median. In general, more than half the ratios will fall within one COD of the median.

The COD should not be calculated about the mean, because the mean is more affected by extreme ratios than the median and because of the inherent bias of the mean (see section 7.3.2). The COD also should never be calculated about the weighted mean, which implicitly weights each ratio based on its sale price (see section 7.3.3).

7.4.2 Coefficient of Variation

The coefficient of variation (COV) can be another important measure of appraisal uniformity. The COV for a sample is calculated by (1) subtracting the mean from each ratio, (2) squaring the calculated differences, (3) summing the squared differences, (4) dividing by the number of ratios less one to obtain the "variance," (5) taking the square root to obtain the "standard deviation," (6) dividing by the mean, and (7) multiplying by 100. Note that the COV is calculated only about the mean—not the median or weighted mean (although other methods permit calculation about the weighted mean). For the data in table 2,

$$\text{variance} = 3.0808 \div 35 = 0.0880;$$

$$\text{standard deviation} = \sqrt{0.0880} = 0.2966;$$

$$\text{COV} = (0.2966 \div 0.900) \times 100 = 33.0.$$

The interpretation of the standard deviation and COV rests on the assumption that the ratios are normally distributed. When this is the case, approximately 68 percent of the ratios will lie within one standard deviation of the mean, and approximately 95 percent will lie within two standard deviations of the mean. When the ratios do not approximate a normal distribution, these relationships no longer hold (although there always will be at least 75 percent of the ratios within two and at least 89 percent of the ratios within three standard deviations of the mean). Hence, one should determine whether ratios are approximately normally distributed before using the COV. When the normality assumption is met, the COV provides the most precise measure of variability. Because the deviations between each ratio and the mean ratio are squared in determining the COV, ratios that differ greatly from the mean influence the COV more than they do the COD, in which the deviation of each observation from the median is equally weighted.

7.4.3 Other Measures of Variability

Other useful measures of variability or the distribution of ratio study data include the

- range
- percentiles
- quartiles
- interquartile range
- median absolute deviation
- median percent deviation
- coefficient of concentration
- weighted coefficient of dispersion
- weighted coefficient of variation

(See [IAAO 1990, chapter 20] and Gloudemans [1999, chapter 5]) for a discussion of these measures).

Note that the typical percentage misassessment is not the COD, but is the median percentage deviation. Also it is the interquartile range, not the COD, that brackets 50 percent of the assessment errors. Finally, various measures of concentration state what percentage of the sample falls within a specified distance of a measure of central tendency.

7.5 Measures of Reliability

Reliability, in a statistical sense, concerns the degree of confidence one can place in a calculated statistic for a sample. (For example, how accurately does the sample median ratio approximate the true [population] median appraisal ratio?) There are two related measures of reliability: confidence intervals and standard errors. A confidence interval consists of two numbers that bracket a calculated measure of central tendency for the sample; one can have a specified degree of confidence that the true measure of central tendency for the population falls between the two numbers. Standard errors relate to the distance one must add to and subtract from certain measures of central tendency to compute the confidence interval.

For the data in table 2, the 95 percent confidence interval for the median is 0.684 to 1.067 (calculations not shown)—from the sample data, one can be 95 percent confident that the median level of appraisal for the population is in this range. New computer-intensive statistical methods, such as the bootstrap (Efron and Tibshirani 1993), now enable the development of confidence interval estimates for any statistic of interest. Confidence intervals can be calculated about various measures of level and uniformity or about a resulting property value estimate (Sherrill and Whorton 1991); standard errors can properly be calculated about the mean and weighted mean, or about an estimate of value for the population. (See IAAO [1990, chapter 20] and

Gloudemans [1999, chapter 6]) for information on performing these calculations.)

Measures of reliability explicitly take into account the errors inherent in a sampling process. In general, these measures will be tighter (better) when samples are relatively large and the uniformity of ratios is relatively good. Although the mathematics of calculating these measures is comparatively straightforward, their correct interpretation is critical and requires someone well grounded in the underlying statistical principles.

It is good practice to calculate measures of reliability whenever the results of a ratio study will be used for equalization. Measures of reliability will indicate whether one can have a desired degree of confidence that a given level of appraisal has *not* been achieved. This does not mean that one should be willing to tolerate measures of central tendency that fail to meet official requirements whenever measures of reliability are wide due to small samples, poor uniformity, or both. Such cases require either additional data for proper analysis or alternative action, perhaps a reappraisal, if poor uniformity is the cause. However, equalization adjustments may not be warranted in these cases because, statistically, the proper assumption is that the appraised value is correct.

The burden of proving that the level of appraisal is not correct and that an equalization adjustment is necessary should be on the entity with equalization authority. When confidence intervals overlap into ranges of acceptability, equalization cannot be supported statistically. By the same token, when confidence intervals *fail* to bracket official requirements, equalization actions are supported.

Uniform assessments result in narrower confidence intervals and thereby increase the chance that a stratum can actually be proven in or out of compliance with standards. Oversight agencies should work with local jurisdictions to correct poor uniformity. Such correction might include reappraisal, appropriate selective trending, and respecifying and recalibrating mass appraisal models. (See section 14 for a discussion of ratio study standards.)

7.6 Vertical Inequities

The measures of variability discussed in section 7.4 relate to "horizontal," or random, dispersion among the ratios in a stratum, regardless of the value of individual parcels. Another form of inequity may be systematic differences in the appraisal of low-value and high-value properties, termed "vertical" inequities. When low-value properties are appraised at greater percentages of market value than high-value properties, assessment *regressivity* is indicated. When low-value properties are appraised at smaller percentages of market value than high-value properties, assessment *progressivity* results. Appraisals made for tax purposes, of course, should be neither regressive nor progressive.

An index statistic for measuring vertical equity is the price-related differential (PRD), which is calculated by dividing the mean by the weighted mean:

$$\text{Mean/Weighted mean} = \text{Price-related differential.}$$

This statistic should be close to 1.00. Measures significantly above 1.00 tend to indicate assessment regressivity; measures below 1.00 suggest assessment progressivity. For the data in table 2, the PRD is 0.983, suggesting slight progressivity. When samples are small or the weighted mean is heavily influenced by several extreme sales prices, however, the PRD may not be a reliable measure of vertical inequities. If not representative, extreme sales prices may be excluded in calculation of the PRD. Similarly, when samples are very large the PRD may be too insensitive to show small pockets where there is significant vertical inequity. Standards for evaluating the PRD are given in section 14.2.7. In addition, statistical tests for vertical inequities are available and should be employed to determine the significance of the indication provided by the PRD (see section 7.7).

When these tests show vertical inequities, such inequities should be addressed through reappraisal or other corrective actions. In some cases, additional stratification can help isolate the problem. Measures of level computed for value strata should not be compared as a way of determining vertical inequity because of a boundary effect that is most pronounced in the highest and lowest strata (Schultz 1996).

7.7 Tests of Hypotheses

An appropriate test should be used whenever the purpose of a ratio study is implicitly or explicitly to test a hypothesis. A hypothesis is essentially a tentative answer to a question, such as: Are residential and commercial properties appraised at equal percentages of market value? A test is a statistical means of deciding whether the answer "yes" to such a question can be rejected at a given level of confidence. In this case, if the test leads to the conclusion that residential and commercial properties are not appraised at equal percentages of market value, some sort of corrective action on the part of assessing officials is clearly indicated.

From this example, it can be seen that hypothesis testing has considerable practical value in assessment administration when there are observed differences in overall appraisal ratios. If the differences can be attributed to chance (sampling error), then such corrective actions as ordering reappraisal, issuing equalization orders, and the like may be inappropriate. Similarly, when the observed level of appraisal for a stratum of properties is below a required standard, an appropriate test can be made to determine whether the difference can be attributed to sampling error (confidence intervals could also be used for the same purpose).

Tests are available to determine whether ratios can be regarded as normally distributed, whether the level of appraisal of a stratum fails to meet an established standard, whether meaningful differences exist in the level of appraisal between two or more strata, and whether high-value properties are appraised at a different percentage of market value than low-value properties. Appropriate tests are listed in table 5 and discussed in Gloudemans (1999), IAAO (1990) and IAAO (1978, 137–54).

7.8 The Normal Distribution

Although ratio data will seldom conform closely to a normal distribution, an effort should be made to investigate the underlying distribution before interpreting uniformity statistics. There are numerous statistical tests available to detect a nonnormal distribution. Some of the traditional choices are the binomial, chi-squared, and Lilliefors tests. Newer and more powerful tests based on the empirical distribution function, skewness, and kurtosis include the Anderson-Darling A^2, Shapiro-Wilk W, and the D'Agostino-Pearson K^2.

7.8.1 Parametric and Distribution-free (Nonparametric) Statistics

For every problem that might be solved by using statistics, there is usually more than one measure or test. These measures and tests can be divided into two broad categories: parametric and distribution-free (nonparametric).

Parametric statistics make certain assumptions about the distribution of the population ratios or functions of the population ratios (mean, weighted mean, and so on). That is, in regard to mean or weighted mean ratios, if the sample size is large enough (approximately thirty) and the sample is representative of the population, the distribution of sample means is approximately normally distributed regardless of the distribution of the individual ratios (Central Limit Theorem). When these assumptions are met, parametric statistics make more efficient use of the data than distribution-free statistics.

Distribution-free statistics make less restrictive assumptions and don't require assumptions about the shape of the population distribution. Distribution-free statistics include the median and the coefficient of dispersion. For a discussion of parametric and distribution-free tests useful in ratio studies, see sections 7.3–7.7, IAAO (1990, chapter 20), Gloudemans (1999, chapter 6), and Conover (1980).

Although the choice between parametric and distribution-free statistical measures and tests can often be important, it should be noted that proper study design, particularly with respect to representative sampling, stratification, and sales screening, will increase the reliability of both parametric and distribution-free measures.

8. Sample Size

8.1 Importance of Sample Size

There is a general relationship between statistical precision and the number of observations in a sample drawn from a given population: the larger the sample, the greater the precision. The required sample size for any given degree of precision depends primarily on accept-

Table 5
Tests of Hypotheses

Null hypothesis	Nonparametric test	Parametric test
1. Ratios are normally distributed.	Binomial test Chi-square test Lilliefors test D'Agostino-Pearson K^2 test Anderson-Darling A^2 test Shapiro-Wilk W test	N/A
2. The level of appraisal meets statutory requirements.	Binomial test	t-test
3. Two property groups are appraised at equal percentages of market value.	Mann-Whitney test	t-test
4. Three or more property groups are appraised at equal percentages of market value.	Kruskal-Wallis test	F-test
5. Low- or high-value properties are appraised at equal percentages of market value.	Spearman Rank test Correlation or regression analysis	
6. Sold and unsold parcels are treated equally.	Mann-Whitney test	t-test

Table 6
Confidence Intervals and Sample Size:
95 percent confidence interval

Sample size	COV = 10.0	COV = 20.0	COV = 30.0
5	±12.4	±24.8	±37.2
10	±7.2	±14.3	±21.5
50	±2.8	±5.5	±8.3
100	±2.0	±3.9	±5.9
300	±1.1	±2.3	±3.4

able sampling error and the variability in the population. When there are insufficient sales to achieve target levels of precision, all valid sales should be used unless this results in nonrepresentativeness. If sales are abundant, sufficient sales to obtain uniform or reasonably narrow margins of error may be randomly selected. If it is desirable to create narrow, uniform margins of error in jurisdictions without sufficient sales, independent appraisals may be added.

8.2 Evaluating the Adequacy of a Given Sample Size

One can evaluate the adequacy of a given sample size by computing measures of reliability (see section 7.5). If the standard error or confidence interval is sufficiently small, the sample is large enough. If the standard error or confidence interval is too wide, the analyst will either have to accept less precision or collect additional observations. Table 6 shows 95 percent confidence intervals for selected sample sizes and coefficients of variation, assuming a normal distribution.

8.3 Required Sample Size

Formulas are available to compute the minimum sample size necessary to produce selected margins of error at a specified level of confidence. See Cochran (1971), Sherrill and Whorton (1991), and Gloudemans (1999) for formulas for stratified and unstratified sample sizes and a correction factor to be used when the population is small. In small or rural jurisdictions, the remedies suggested in section 8.4 may have to be applied to achieve the required sample size.

8.4 Remedies for Inadequate Samples

Small samples should be enlarged, if possible, where operational requirements dictate that there be a reliable estimate of the level of appraisal, such as for equalization purposes. Inadequate sample sizes are typically indicated by unacceptably wide confidence intervals (see section 7.5). The following alternatives should be considered:

1. *Restratification.* If circumstances permit, broader strata containing larger samples can be created by combining existing strata or by stratifying on a different basis. However, this approach may also increase the variability of the ratios in the resulting strata. Often it is desirable to create two or more levels of strata, with the more detailed strata used primarily for purposes of discerning patterns of appraisal performance within the primary strata.

2. *Extending the period from which sales are drawn.* This is often the most practical and effective approach. Unless property values are changing rapidly, sales up to three years old can be used in ratio studies, provided time adjustments are considered and made as necessary. In cases where sales are particularly limited, sales up to five years old can be used if severe economic shifts have not occurred and if time adjustments are carefully considered. However, these sales may not provide useful information if a revaluation has occurred after the sale and the independence between appraisals and sales prices cannot be verified. The longer the time period, the more necessary are tests for independence between appraised values and sales prices. Additional statistical tests should be conducted to confirm that no significant differences exist between the performance measures for supplemental sales under consideration and the sample of recent sales.

3. *Enlarging the sample by validating previously rejected sales.* Sales that were previously excluded from the analysis because it was not administratively expedient to confirm them or to make adjustments for personal property, financing, and the like can be reevaluated. If previously rejected sales are included, additional efforts to confirm them or adjust them should be made.

4. *Using appraisals in lieu of or in addition to sales prices.* This alternative is discussed in section 9 below.

5. *Imputing appraisal performance.* Ratio study statistics for strata with no or few sales can sometimes be imputed from the results obtained for other strata. These strata should be as similar as possible. Procedures and techniques used to appraise properties in the strata should also be similar.

6. *Assuming statutory level of assessment.* If the alternatives for enlarging the sample fail and it is not considered appropriate to impute results from other strata, any stratum that has not been subject to ratio study analysis and constitutes no more than 5 percent of the value of a jurisdiction may be assumed to be at the statutory level of assessment. If the stratum includes more than 5 percent of the value of a jurisdiction, the equalization agency should conduct appraisals or procedure audits to determine the appropriate level of assessment.

8.5 Other Sample Size Problems

Sales from areas or substrata where the number of sales is disproportionately large can distort ratio study results by weighting level and uniformity indicators toward what-

ever conditions exist in the over-represented area. To alleviate this problem and create better representativeness, samples can be "thinned" by randomly selecting sales to be removed. The same process can be used to standardize margins of sampling error to produce, for example, confidence intervals of more uniform width. Alternatively, the overrepresented area can be treated as a substratum by applying its ratio only to those properties in that area.

9. Appraisal Ratio Studies

Few ratio studies are based solely on independently conducted appraisals, which then are compared to values determined by assessing officials. Many equalization or oversight agencies, however, do ratio studies in which both sales and appraisals are combined.

9.1 Rationale

If there are not enough sales to provide the necessary representativeness or precision, independent appraisals can be used as indicators of or surrogates for market value. Independent appraisals are appraisals performed by appraisers who are not employees of the appraisal agency that is the subject of the study. Such appraisal ratio studies can be particularly useful for commercial and industrial real property and personal property for which sales data are limited (see IAAO [1990, appendix 1-1] and Gloudemans [1999, chapter 6]). In addition, appraisal ratio studies can be used for agricultural or other properties not appraised on an ad valorem basis. In this case, the appraisals should reflect the use value or other statutory basis on which the properties are appraised.

9.2 Advantages and Disadvantages

Appraisal ratio studies have both advantages and disadvantages. The major advantage is a high degree of control in sample selection. One can specify the size of the sample and ensure that it is drawn randomly, thus helping to ensure representativeness. If objectivity can be maintained, the appraisal ratio study avoids potential distortions due to systematic differences between appraisals of sampled and unsampled properties. In addition, independent appraisals can be used to test for systematic differences between appraisals of sold and unsold properties.

A major disadvantage of appraisal ratio studies is the time and cost involved. The subject and any comparables should be physically inspected and the appraisals carefully documented. Per-parcel costs of such appraisals are necessarily high. Although the level of detail and presentation need not be as comprehensive as a narrative single-property appraisal, applicable *USPAP* guidelines should be followed. The performance of a Limited Appraisal is often appropriate, and preparation of a Restricted Appraisal Report, as defined under Standard Rule 2-2(c), is typically adequate. Independent appraisals done with a mass appraisal model should comply with Standard 6.

Another major disadvantage is that appraisals are an opinion of value and therefore necessarily subjective. Accordingly, they should be documented and tested against the market. However, this becomes difficult when sales data are scarce.

9.3 Sample Selection and Resource Requirements

Sample selection in an appraisal ratio study requires knowledge of statistical principles and available resources. Judgment must be used, because the determination of an adequate sample may require more information than is available, such as the probable variability of the ratios. Moreover, the cost of the study will increase with the size of the sample. Therefore, the value of more accurate information must be balanced against the costs of obtaining that information.

The size of the sample for each stratum should be determined with consideration given to (1) the required precision of the estimate of the appraisal level, for example, plus or minus .05; (2) the required confidence level, for example, 95 percent; (3) the expected amount of dispersion in the ratios; and (4) the wastage associated with properties that cannot be efficiently appraised or appraisals that cannot be used for one reason or another. (See Gloudemans [1999, chapter 6] for sample size formulas).

Once the size of an appraisal sample has been determined, the individual properties that will constitute the sample should be selected using a statistically valid sampling plan.

9.4 Data Requirements and Appraisal Techniques

The appraisal techniques selected for an appraisal ratio study should be consistent with accepted appraisal principles and practices. The appraisals should reflect the appraisal date in question and should be well documented. Computers should be used as much as possible to expand analytical capabilities and simplify calculations.

The appraisals used in appraisal ratio studies can be based on computer-assisted mass appraisal (CAMA) techniques. This helps ensure the objectivity of the appraisals and facilitates quality control. The models used, however, must be developed independently from those used for assessment purposes. If available, sales from a later period should be used to ensure independence. CAMA models have the additional advantage of reducing parcel costs, permitting the use of larger, more representative samples. Adequate and well-documented market data are required to develop reliable and defensible model estimates for the three approaches to value.

Because the purpose of the appraisal is to make an *independent* value estimate, not audit the assessor's work,

appraisers should *not* be supplied with copies of the assessor's appraisal work sheets or model information. When the purpose of the ratio study is equalization or performance measurement, rather than internal quality assurance, the appraisals should not be revealed to the assessor until the assessor's values are final.

9.5 Reviewing the Appraisals

Appraisal models or individual appraisals (where single-property appraisal techniques are used) should be reviewed by supervisory appraisers of the agency making them to ensure that *USPAP* and the agency's standards are met. It is also good practice to include some recently sold properties in the sample being appraised as a check on the validity of the methods being applied. In addition, the assessor should be afforded an opportunity to review the appraisals along with supporting documentation and to submit information supporting different value conclusions. If different value conclusions would materially affect the outcome of the study, a procedure for resolving conflicts, for example, by an independent review body, should be established.

9.6 Combining Sales and Appraisals

Appraisals can be combined with valid sales in a ratio study. Whether or not such an approach should be followed, however, requires careful analysis and judgment. Using available sales adds objectivity to the study and reduces the required number of appraisals. On the other hand, combining sales and appraisals mixes two market indicators. If sales and appraisals are combined, an analysis should be performed to ensure that the sales and appraisal ratios are consistent with respect to their measures of central tendency. A Mann-Whitney test comparing values per unit or comparing ratios based on sales with those based on appraisals is appropriate for this purpose. Significant differences often indicate that the appraisals do not reflect the actual market or that unsold properties may be assessed differently from those that sell (see section 10).

Variability measures computed on sales used in the sample should not be expected to be similar to variability measures computed on appraisals. Sales ratios reflect the vagaries of the marketplace. Appraisal ratios, on the other hand, come from comparing the results of one appraisal model (the oversight agency's) to the results of another (the assessing office's). If both parties use mass appraisal procedures, differences in appraisals between the two models should be less than when compared with sales; thus, variability measures based on appraisal ratios can be expected to be less than those based on sales ratios.

9.7 Average Unit Value Comparisons

In addition to a traditional ratio study, "expert" appraisals can take the form of average unit values and be compared against the assessor's average unit value for the same parcels. In this technique, parcels are stratified into homogeneous groups as they would be for appraisal purposes. Appropriate units of comparison are identified for each stratum, and average unit values are determined through an analysis of available sales, cost, and income data. The assessor's average unit values for the same strata are then calculated and the two averages are compared. Although this technique lacks measures of dispersion and statistical reliability, it is well-rooted in mass appraisal theory and offers an alternative to the time and expense associated with the selection and appraisal of individual parcels.

10. Estimating Performance for Unsold Properties

10.1 Rationale

The objective of a ratio study is to determine appraisal performance for the population of properties. As long as sold and unsold parcels are appraised in the same manner, statistics calculated in a sales ratio study can be used to infer appraisal performance for unsold parcels. However, if parcels that sell are selectively reappraised based on their sales prices or some other criterion (such as listing price) , sales ratio study uniformity measures will not be valid (appraisals will appear more uniform than they are). In this situation, measures of appraisal level will also be invalid unless similar unsold parcels were reappraised by a method that produces the same percentage of market value (appraisal level) as on the parcels that sold. Assessing officials must ensure that sold and unsold parcels are treated equally. Several techniques are available for determining whether assessors are selectively appraising sold parcels (see IAAO [1990, appendix 20-2] and Gloudemans [1999, chapter 6]) for a more detailed discussion).

If unsold properties are not appraised consistently with sold properties and applicable guidelines, unadjusted sales ratio results cannot be used. The oversight agency will have to adjust calculated results or conduct an alternative study (see section 10.7). In addition, the appraisal agency should be ordered to cease the unprofessional practice of sales chasing and to reappraise all property on a consistent basis using applicable standards and guidelines.

10.2 Comparison of Average Value Changes

If sold and unsold properties are appraised in the same way, their appraised values should change from year to year in a similar manner. Accordingly, one can compare changes in appraised values for sold and unsold parcels to discern whether sold parcels have been selectively appraised.

For example, if values for sold parcels in a given stratum increased an average of 10 percent while values for unsold parcels in the same stratum increased an average of only 2 percent, "sales chasing" probably exists. At a more sophisticated level, one can compare the distribution of value changes for sold and unsold parcels or use statistical tests to determine whether the distributions are different at a given level of confidence (see table 5).

10.3 Comparison of Average Unit Values

If sold and unsold parcels are appraised equally, average unit values (for example, value per square foot) should be similar for parcels in the same stratum. An appropriate test (Mann-Whitney or *t*-test) can be conducted to determine whether differences are significant.

10.4 Split Sample Technique

In this technique, two ratio studies are performed, one using sales that occurred before the appraisal date, and one using sales after the appraisal date, both adjusted for date of sale as appropriate. Except for random sampling error, results of the two studies should be similar. Sales chasing is indicated if the results of the first study are consistently better than those from the second. In such a case, the second study is still valid; the first study should be rejected.

10.5 Mass Appraisal Techniques

Provided they are sufficient in number, sales used in ratio studies can be used to develop mass appraisal models to apply against a random sample of unsold properties. Multiple regression or another automated form of the sales comparison approach can be used to develop the models. An appraisal ratio study is then conducted for the random sample of unsold parcels using values predicted by the models as surrogates for market values. This approach has the following advantages:

1. It is objective and rooted in the market.

2. The models can be reviewed for reliability before being applied to the random sample of unsold parcels.

3. The technique yields measures of central tendency, which can be compared against those produced by the sales ratio study and tested for compliance with standards for the level of appraisal.

4. The technique takes the form of an appraisal ratio study but avoids the time and expense of narrative appraisals.

Reliability of this method depends on the accuracy of the mass appraisal models used to generate the value estimates. The models should be consistent with appraisal theory and reviewed for reliability by examining goodness-of-fit statistics. The models should be independent of those used for assessment purposes.

10.6 Comparison of Observed vs. Expected Distribution of Ratios

It is possible to obtain a strong indication of the likelihood of "sales chasing" by computing the proportion of ratios that would be expected to fall within a particular narrow range of the mean given the lowest likely standard deviation (although this depends somewhat on the assumption of a normal distribution). For example, with a standard deviation of 5 percent (virtually unachievably uniform ratios) given a normal distribution, one would expect to find about 32 percent of the ratios within ±2 percent of the mean (for example, between 98 and 102 percent, given a mean of 100 percent). Regardless of the distribution of the ratios, the likelihood is extremely low that there would be a representative sample with more than this proportion of ratios in such a narrow range. If such is the case, "sales chasing" is the most likely conclusion.

10.7 Solutions to "Sales Chasing"

Once it is determined that "sales chasing" probably has occurred and probably is reducing the validity of ratio study statistical measures of level or uniformity, it is necessary to redo the ratio study to establish valid measures before any other recommendations, such as reappraisal or equalization action, can be made. If feasible, probably the best approach is to select a sample period that effectively precludes sales chasing. For example, when the lien or appraisal date is January 1, many jurisdictions will use sales occurring before that date to make valuation decisions. To test the resulting valuations, it would be appropriate to use sales occurring after January 1, provided such data are time-adjusted (when necessary) backwards to match the appraisal date. As a slight variation on this principle, it may be possible to use earlier sales in most cases, but when "sales chasing" is detected, to switch to a later, post-appraisal-date sales period.

Statutory or practical constraints may prevent use of optimal sample periods in many cases. In these situations, it is important to determine the exact cause of the "sales chasing." For example, if a large proportion of selling properties are appealed and if appeal boards typically adjust to sale price, the result will be the same as "sales chasing" by the assessor. One solution is to use appraised values prior to the action of the appeal board, provided that the appeal adjustment is not merely the result of an atypical clerical or other error.

Another approach is to use current sales prices and prior year's values, adjusted for reappraisal activity, and prior year's appraised values, adjusted for reappraisal activity or assessment value changes in the population. The percentage increase or decrease in the prior year's appraised values for the population (net of new construction) should be used to adjust the prior year's values for the sample (Gloudemans 1999). See section 5.3.

11. Personal Property Studies

Horizontal equity requires similar levels of appraisal between real and personal property. Sales data for personal property are difficult to obtain and analyze because markets for personal property are generally less visible and more difficult to follow than real property markets. Therefore, performance reviews and appraisal ratio studies should be used in place of sales ratio studies to determine the quality of appraisal

of personal property. The performance review does not quantify assessment conditions but can determine general assessment quality. The appraisal ratio study can be used to determine the level and uniformity of assessment for personal property (Wheeler and Cornia n.d.; Daw 1989).

11.1 The Performance Review

The performance review is an empirical study that evaluates the assessment method used and the ability of the jurisdiction to meet its statutory requirement in the assessment of personal property. This type of study may be used to allocate tax dollars in multijurisdictional funding calculations or equalization by assuming that jurisdictions passing the performance review are assessing personal property at the general level of other classes of property analyzed with ratio studies.

The study is completed by determining the amount of resources directed toward the assessment of personal property and reviewing appraisal and discovery methods.

11.1.1 Personnel

One employee should be assigned for every 2,000–4,000 accounts. The appropriate number depends on the complexity of the requirements, that is, inclusion of intangibles, inventories, household goods, agricultural products, motor vehicles, and complex exemptions. The appropriate number is also influenced by the amount of assistance provided by state or regional agencies.

11.1.2 Discovery

The jurisdiction must have the ability to discover the owners or users of taxable personal property within the jurisdiction. This is accomplished using phone books, business/occupational licenses, listings, sales tax rolls, and field reviews (see IAAO Course 500, The Assessment of Personal Property, and *Standard on Personal Property Valuation* [IAAO 1996] for a complete list).

11.1.3 Valuation

Personal property is valued using acceptable schedules and methods including depreciation schedules published by nationally recognized valuation firms, market data from published valuation guides, and other generally accepted valuation methods.

11.1.4 Verification

Inclusiveness of personal property returns and reports should be verified by an audit program. The audit program should focus on larger and problem accounts. However, it should also include randomly selected accounts. The audit program should provide coverage of the entire tax base irrespective of the jurisdiction's reappraisal cycle.

11.1.5 Forms and Renditions

Comprehensive forms supplied by the assessment authority should allow the taxpayer to disclose fully all assessable personal property. The tax laws should require mandatory compliance, with meaningful penalties for noncompliance.

Evidence of the overall thoroughness of the jurisdiction may be indicated by the number of value change notifications and penalties imposed during the personal property assessment process.

11.2 Appraisal Ratio Studies

The appraisal ratio study is a numerical study that produces an estimate of the level of assessment of personal property by developing a ratio of assessment for property that is on the tax roll through the use of appraisals. The level of assessment determined in this way must be adjusted downward to account for property that has not been assessed.

11.2.1 Determining the Assessment Ratio for Personal Property

Personal property market values are usually derived from appraisals using a replacement cost new less depreciation (RCNLD) approach (see IAAO Course 500). A comparison of the depreciation schedules in use to nationally accepted schedules would enable the calculation of a ratio for property on the roll. A statistically sound process should be used to select a sample that is representative of personal property on the tax rolls. Such a sample may be parcel- or value-based depending on the intended use of the ratio study in indirect or direct equalization.

11.2.2 Stratification

Proper stratification of personal property accounts should be done for greater statistical reliability. (See section 4.4). Strata should be based on the type and value of personal property accounts.

Stratification by type of account should occur first. Personal property accounts can be divided into residential (motor vehicles, boats, aircraft, etc.), agriculture, and business accounts. Further stratification may occur in residential and agricultural accounts but must occur in business or commercial accounts. Business accounts are usually stratified by size into a minimum of four groups, Value ranges for these groups should be derived from the value ranges in the local market. One example would be small (less than $250,000), medium ($250,000 to $1 million), moderate ($1 million to $5), and large (greater than $5 million). Individual size of account may be determined by value on the previous year tax roll. Typically, about 20 percent of the business personal property accounts (moderate and large groups) account for approximately 80 percent of the business personal property value.

11.2.3 Property Escaping Assessment

Personal property is particularly prone to escape assessment. Some determination should be made about the portion of taxable personal property not on the assessment roll. However, estimates based on national averages are virtually meaningless and should not be used.

11.2.3.1 Identifying Personal Property Owners and Users Not on the Roll

Discovery tools (see section 11.1.2) can be used to determine accounts not on the roll for a sample area or group. Once the extent of the problem is identified, a projection can be made of the percentage of personal property not identified on the assessment roll.

11.2.3.2 Identifying Personal Property Not Included in Taxpayer Returns/Reports

The accepted method of determining the property omitted in taxpayer returns/reports is to audit the account (see IAAO Workshop on Auditing). The audit results are applied back to the account value. The resulting fraction is property escaping taxation within that particular personal property account. If appropriate sampling techniques are used in selecting the accounts for audit, the resulting ratio would be applied to the total roll to help determine the percentage of personal property escaping assessment within the jurisdiction.

11.2.4 Computing the Level of Appraisal

The overall ratio is then determined by reducing the valuation ratio by the percent of property wholly or partially escaping taxation. Example: If the appraisal level were found to be 89.4 percent and 6.1 percent of property is escaping assessment, then the corrected level of assessment is the appraisal level times the percent of property assessed $0.894 \times (1 - 0.061) = 0.839$.

12. Presentation of Findings, Documentation, and Training

The findings of a ratio study should be detailed enough to meet the needs of the users of the study. The credibility and usefulness of the study will depend largely on the details provided and the clarity of the presentation. The frequent misuses and misunderstandings of ratio studies can be reduced by providing interpretive information. The following should be provided in conjunction with a ratio study:

12.1 Text

A text describing the purpose, the authority for (if applicable), and the methods used should accompany a ratio study. Whether this information should be incorporated in the report of the findings or be contained in a separate memorandum, report, or manual will depend primarily on the audience of the study. The text need not be a detailed procedural manual; rather it should contain definitions of key terms, including the statistics presented, and outline the major procedural steps in completing the study. The text should also describe any rules for eliminating sales or extreme ratios and acknowledge limitations in the data. Detailed procedural manuals (see section 12.4) should be available for inspection by concerned parties.

12.2 Exhibits

The body of the ratio study should include for each stratum the statistical results intended to be used for decision-making purposes. All reports should be dated and indicate the tax year of the appraisals being evaluated and the period selected for sales used in the study. It is good practice to show the number of parcels in each stratum, the number of sales and appraisals in the period selected, the number of sales and appraisals used in the study, several measures of central tendency, one or more measures of uniformity, the PRD, standard error or confidence interval about the primary measure of central tendency, and a summary of adjustments made to sales prices.

Data displays (see section 7.2) should be included to illustrate key statistics. All reports, graphs, and other exhibits should be clearly labeled.

12.3 Analyses and Conclusions

An objective statement of the results of the ratio study should be prepared. If the study is one in a series, a comparison of the results with those of previous studies can be helpful. The summary statement might be included in a cover letter to interested parties.

12.4 Documentation

Ratio study procedures should be thoroughly documented. This documentation should take three forms. First, there should be a procedural manual that explains the design of the study; data collection, confirmation, and screening procedures; adjustments for financing, personal property, and time (if applicable); calculation of statistics used in the study; and other relevant procedures. This manual should be updated whenever procedures are changed. Second, the software should be documented so that the program logic can be reviewed and modified as needed. Third, there should be a user's manual that explains how to run the software, how to specify sale dates and other parameters, how to produce the desired number of copies, and so forth. It is unwise to rely solely on the knowledge of one key person to maintain a ratio study system.

12.5 Training and Education

The effectiveness of ratio studies can be improved through education and training. Supervisory agencies should consider conducting seminars or workshops that explain their studies, how to interpret the reports, how ratio studies can be used to improve appraisal procedures and performance, and so on. Without adequate guidance, local assessors may not realize that they are in potential conflict with valuation standards and ratio study requirements. More important, education about the methods of ratio studies will promote understanding and proper use of the studies, as well as technical improvements.

13. Computer Options

Ratio studies should be performed with computers to ensure accuracy and achieve efficiency.

13.1 Hardware

Hardware should be sufficient to support the objectives of the study. Assessment agencies should have available enough primary storage to process all sales data to be used in the study simultaneously, so that measures of central tendency and dispersion can be calculated on a jurisdictionwide basis. Personal computers with sufficient speed and storage are adequate for performing ratio studies. When data are stored on a mainframe computer or server, the ability to download data for analyses using personal computers is highly desirable.

13.2 Software

Three basic software options are (1) purchase of special-purpose commercial software, (2) development of in-house software, and (3) adoption of general-purpose software. Each has relative advantages and disadvantages. If ratio study software is to be purchased, the user should ensure that it calculates the required statistics and possesses adequate flexibility. Software developed in-house tends to be relatively expensive and difficult to modify, but can be written to meet the user's specific requirements. The adoption of general-purpose software offers a compromise between the first two options. Such software is flexible, pretested, easy to modify, and relatively inexpensive. Adapting it to ratio studies, however, requires someone skilled in use of the software as well as in ratio study design. Regardless of the method chosen, the system should be well-documented and more than one person should be familiar with its execution.

For more information, see the IAAO *Standard on Contracting for Assessment Services* (1986), the IAAO *Standard on Facilities, Computers, Equipment, and Supplies* (1996), IAAO (1990), and Gloudemans (1999).

13.3 Data Integrity

Accuracy and integrity of data entered into or transferred through computer systems must be ensured. Design of computer programs should make it easy to verify data accuracy. Queries should be easily accessible to data entry personnel or other users, so that data can be verified without continual reliance on computer programming staff. Methods to check accuracy of assigned strata (such as school district, city neighborhood, and category) as well as of assessed or appraised value, sale price, parcel identifier, and other fields must be established and made user friendly.

14. Ratio Study Standards

Each state, province, and local jurisdiction should have ratio study performance standards. Local standards should be consistent with state or provincial standards. The following standards, summarized in table 7 are suggested for jurisdictions in which current market value is the legal basis for assessment. In general, when these standards or other local standards are not met, reappraisal or other corrective measures should be taken, or equalization procedures imposed. When an oversight agency orders such actions, the burden of proof should be on the agency to show that the standards have not been achieved.

14.1 Level of Appraisal

The overall level of appraisal of the jurisdiction and each major class of property (such as residential, commercial/industrial, and vacant land) should be between 0.90 and 1.10 (within ±10 percent of the statutorily required level of assessment), or 0.95–1.05 for indirect equalization (within ±5 percent of the statutorily required level of assessment) although jurisdictions may set more stringent standards. By themselves, the calculated measures of central tendency provide only an indication, not proof, of whether the chosen standard has been met. Confidence intervals and statistical tests (see section 7.5)

Table 7
Ratio Study Performance Standards

Type of property	Measure of central tendency	COD	PRD*
Single-family residential			
Newer, more homogenous areas	0.90–1.10	10.0 or less	0.98–1.03
Older, heterogeneous areas	0.90–1.10	15.0 or less	0.98–1.03
Rural residential and seasonal	0.90–1.10	20.0 or less	0.98–1.03
Income-producing properties	0.90–1.10		
Larger, urban jurisdictions	0.90–1.10	15.0 or less	0.98–1.03
Smaller, rural jurisdictions	0.90–1.10	20.0 or less	0.98–1.03
Vacant land	0.90–1.10	20.0 or less	0.98–1.03
Other real and personal property	0.90–1.10	Varies with local conditions	0.98–1.03

*The standards for the PRD are not absolute when samples are small or when wide variations in prices exist. In such cases, appropriate tests are more useful (see table 5).

should be used to determine whether one can reasonably conclude that the chosen standard has *not* been met in a particular instance. One can conclude that the standard has *not* been achieved if a confidence interval about the chosen measure of central tendency fails to fall in the required range. In this case, equalization actions are needed. If, however, the calculated level of appraisal fails to fall in the required range, but the confidence interval *does* overlap the required range, one cannot conclude that the standard has not been met. Typically, a conclusion of noncompliance requires a high degree of confidence. It is recommended that 95 percent confidence intervals be required. However, when independent samples show long-term appraisal inequity as evidenced by poor level and uniformity statistics for at least three consecutive years, it is appropriate for the oversight agency to consider using a level of confidence as low as 70 percent.

Table 8 displays changes in confidence interval width associated with differing degrees of confidence. For example, the seventeen sales shown in table 8 have a calculated median of 0.820, and a 95 percent (two-tailed) confidence interval about the median that ranges from 69.30 percent to 94.70 percent. From these data, one *cannot* conclude with 95 percent confidence that a level of appraisal of at least 0.900 has not been achieved, and direct equalization should not be imposed. However, neither should one assume that the standard has been achieved. Using the binomial test, it can be shown that there is only a 7.2 percent probability that the population (true) median is between 0.900 and 1.100.

In the instance described in table 8, there is a strong indication that an adequate level of appraisal has not been achieved. If the number of available sales and median level tends to remain low year after year, further tests using confidence levels as low as 70 percent should be performed. For the sample in table 8, 70 percent, 80 percent, and 90 percent confidence intervals have been computed and indicate that, at confidence levels of 80 percent or less, the true median is unlikely to fall in the desired range of 0.90–1.10. Therefore, despite the 95 percent confidence interval results, direct equalization should be viewed as an appropriate option if the sample median remains low year after year. Alternatively, longer periods from which samples are drawn and added independent appraisals may be considered to produce narrower confidence intervals and reduce margins of error.

14.1.1 Direct Equalization Standards

When direct equalization of locally determined values could result from a finding of non-compliance with standards for appraisal level, the burden of statistical proof of non-compliance should be on the equalizing agency. Furthermore, because appraisal is not an exact science with predetermined precise answers (absolute values), it is recommended that jurisdictions not be found out of compliance with standards for appraisal level unless the ratio study indicates failure to meet the standards presented in this section (statutorily required level ±10 percent) with at least 95 percent confidence (unless a lower level of confidence has been chosen—see 14.1). Thus, the example in section 14.1 would not be found out of compliance and would not be subject to direct equalization under these standards (see section 2.3.2.1). Because of U.S. federally mandated restrictions, such as those found in the 4-R Act, commercial and industrial property and certain utilities may require adjustment when the difference in level provably exceeds ±5 percent.

Table 8
Confidence Intervals at Various Confidence Levels

Rank	Parcel #	Appraised value	Sale price	Ratio	Statistic	Result	Two-tailed confidence intervals using various confidence levels		
1	9	$87,200	$138,720	0.629	Number (n)	17	70% conf. int. mean	0.790	0.864
2	10	38,240	59,700	0.641	Total appraised value	$1,455,330	70% conf. int. median	0.717	0.895
3	11	96,320	146,400	0.658	Total market value	$1,718,220	70% conf. int. wtd. mean	0.803	0.891
4	12	68,610	99,000	0.693	Average appraised value	$85,608			
5	13	32,960	47,400	0.695	Average market value	$101,072	80% conf. int. mean	0.781	0.874
6	14	50,560	70,500	0.717			80% conf. int. median	0.717	0.895
7	15	61,360	78,000	0.787	Mean ratio	0.827	80% conf. int. wtd. mean	0.792	0.902
8	16	47,360	60,000	0.789	Median ratio	0.820			
9	17	56,580	69,000	0.820	Geometric mean ratio	0.816	90% conf. int. mean	0.767	0.888
10	18	47,040	55,500	0.848	Weighted mean ratio	0.847	90% conf. int. median	0.695	0.933
11	19	136,000	154,500	0.880			90% conf. int. wtd. mean	0.775	0.919
12	20	98,000	109,500	0.895	Price-related differential	0.98			
13	21	56,000	60,000	0.933	Coefficient of dispersion	14.5%	95% conf. int. mean	0.754	0.901
14	22	159,100	168,000	0.947	Standard deviation	0.143	95% conf. int. median	0.693	0.947
15	23	128,000	124,500	1.028	Coefficient of variation	17.3%	95% conf. int. wtd. mean	0.759	0.935
16	24	132,000	127,500	1.035					
17	25	160,000	150,000	1.067	Prob 90%–110% population ("true") mean	2.61%			

Date: 0/0/00

When large variability is the cause of a failure to prove that the level of assessment is not within 10 percent of the statutorily required level, reappraisal orders should be considered.

14.1.2 Indirect Equalization Standards

When the purpose of the ratio study is indirect equalization, a more stringent standard should be applied. In this case, it is recommended that funding adjustments be considered whenever the 95 percent confidence interval about the chosen measure of appraisal level fails to fall within 5 percent of the statutorily required level (for example, 0.95–1.05) (see section 2.3.2.2)—unless a lower level of confidence has been chosen. Equalization agencies may choose to adopt a more stringent standard (smaller range) or to equalize whenever the level is provably different from 1.0.

14.1.3 Calculating Equalization Adjustments

If noncompliance with either direct or indirect equalization standards is indicated, the appropriate point estimate (statistic) measuring appraisal level should be used to calculate adjustment factors, by dividing it into 100 percent, for funding or value adjustments (see section 7.3).

14.2 Appraisal Uniformity

Assuming the existence of an adequate and representative sample, if the uniformity of appraisal is unacceptable, reappraisal should be undertaken regardless of the level of appraisal. However, jurisdictions should not be mandated to reappraise unless the ratio study indicates failure to meet the standards presented in this section with an appropriate degree of statistical confidence.

The following uniformity standards are defined in terms of the COD. Approximately equivalent standards of appraisal uniformity can also be developed for the COV when data are normally distributed. In a normal distribution, the COV is approximately 1.25 times the COD. For example, a COD of 10 would roughly correspond to a COV of 12.5.

14.2.1 Uniformity among Strata

Although the goal is to achieve an overall level of appraisal equal to 100 percent of the statutory requirement, ensuring uniformity in appraisal levels among strata is also important. The level of appraisal of each stratum (class, neighborhood, age group, and the like) should be within 5 percent of the overall level of appraisal of the jurisdiction. For example, if the overall level of appraisal of the jurisdiction is 0.90, the level of appraisal of each stratum should be between 0.855 (0.95 × 0.90) and 0.945 (1.05 × 0.90). One can conclude beyond reasonable doubt that this standard has not been met if a 95 percent confidence interval about the chosen measure of central tendency fails to fall within 5 percent of the overall level of appraisal calculated for the jurisdiction (unless a lower level of confidence has been chosen).

14.2.2 Uniformity among Single-Family Residential Properties

The COD for single-family homes and condominiums should be 15.0 or less. In areas of newer or fairly similar residences, it should be 10.0 or less.

14.2.3 Uniformity among Income-Producing Properties

The COD should be 20.0 or less. In larger, urban jurisdictions, it should be 15.0 or less.

14.2.4 Uniformity among Unimproved Properties

The COD for vacant land should be 20.0 or less.

14.2.5 Uniformity among Rural Residential and Seasonal Properties.

The COD for heterogeneous rural residential property and seasonal homes should be 20.0 or less.

14.2.6 Uniformity among Other Properties

Target CODs for special-purpose real property and personal property should reflect the nature of the properties involved, market conditions, and the availability of reliable market indicators.

14.2.7 Vertical Equity

PRDs should be between 0.98 and 1.03. The reason this range is not centered on 1.00 relates to an inherent upward bias in the arithmetic mean (numerator in the PRD) that does not equally affect the weighted mean (denominator in the PRD). When samples are small, have high dispersion, or include properties with extreme values, the PRD may not provide an accurate indication of assessment regressivity or progressivity. Similar considerations apply to special-purpose real property and to personal property. It is good practice to perform an appropriate statistical test for price-related biases before concluding that they exist (see table 5).

14.2.8 Alternative Uniformity Standards

The above standards may not be applicable to properties in unique, depressed, or rapidly changing markets. In such cases, assessment administrators may be able to develop target standards based on an analysis of past performance or results in similar markets elsewhere. Such an analysis can be based on ratio study results for the past five years or more.

15. Definitions

Absolute value. The value of a number (or variable) regardless of its sign. For example, 3 and –3 (minus 3) both have an absolute value of 3. The mathematical symbol for absolute value is one vertical bar on each side of the number in question, for example, |3|.

Adjusted sale price. The sale price that results from adjustments made to the stated sale price to account

for the effects of time, personal property, financing, or the like.

Appraisal. "The act or process of developing an opinion of value; an opinion of value" (*USPAP* 1999). The act of estimating the money value of property. The money value of property as estimated by an appraiser.

Appraisal date. The date as of which a property's value is estimated. *See also* **assessment date.**

Appraisal ratio. (1) The ratio of the appraised value to an indicator of market value. (2) By extension, an estimated fractional relationship between the appraisals and market values of a group of properties. *See also* **level of appraisal.**

Appraisal ratio study. A ratio study using independent expert appraisals as indicators of market value.

Appraisal-sale price ratio. The ratio of the appraised value to the sale price (or adjusted sale price) of a property; a simple indication of appraisal accuracy.

Appraised value. The estimate of the value of a property before application of any fractional assessment ratio, partial exemption, or other adjustments.

Arithmetic mean. A measure of central tendency. The result of adding all the values of a variable and dividing by the number of values. For example, the arithmetic mean of 3, 5, and 10 is 18 divided by 3, or 6.

Array. An ordered arrangement of data, such as a listing of sales ratios, in order of magnitude.

Assessed value. (1) A value set on real estate and personal property by a government as a basis for levying taxes. (2) The monetary amount at which a property is put on the assessment roll for purposes of computing the tax levy. Assessed values differ from the assessor's estimate of actual (market) value for four major reasons: fractional assessment ratios, partial exemptions, preferential assessments, and decisions by assessing officials to override market value.

Assessment. (1) In general, the official act of determining the amount of the tax base. (2) As applied to property taxes, the official act of discovering, listing, and appraising property, whether performed by an assessor, a board of review, or a court. (3) The value placed on property in the course of such act.

Assessment-appraisal ratio. The ratio of the assessed value of a property to an independent appraisal.

Assessment date. The status date for tax purposes. Appraised values reflect the status of the property and any partially completed construction as of this date.

Assessment progressivity (regressivity). An appraisal bias such that high-value properties are appraised higher (or lower) than low-value properties in relation to market values. *See also* **price-related differential.**

Assessment ratio (1) The fractional relationship an assessed value bears to the market value of the property in question. (2) By extension, the fractional relationship the total of the assessment roll bears to the total market value of all taxable property in a jurisdiction. *See also* **level of assessment.**

Assessment-sale price ratio. The ratio of the assessed value to the sale price (or adjusted sale price) of a property.

Assessor. (1) The head of an assessment jurisdiction. Assessors may be either elected or appointed. In this standard the term is sometimes used collectively to refer to all assessment officials charged with administering the assessment function. (2) The public officer or member of a public body whose duty it is to make the original assessment.

Average deviation. The arithmetic mean of the absolute deviations of a set of numbers from a measure of central tendency such as the median. Taking absolute values is generally understood without being stated. The average deviation of the numbers 4, 6, and 10 about their median (6) is $(2 + 0 + 4) \div 3 = 2$. The average deviation is used in computing the coefficient of dispersion (COD).

Bias. A statistic is said to be biased if the expected value of that statistic is not equal to the population parameter being estimated. A process is said to be biased if it produces results that vary systematically with some factor that should be irrelevant. In assessment administration, assessment progressivity (regressivity) is one kind of possible bias.

Bootstrap. A computer-intensive method of statistical inference that is based on a repeated resampling of data to provide more information about the population characteristics. The bootstrap is a data-driven procedure that is particularly useful for confidence interval approximation when no traditional formulas are available or the sample has been drawn from a population that does not conform to the normal distribution.

Central tendency. (1) The tendency of most kinds of data to cluster around some typical or central value, such as the mean or median. (2) By extension, any or all such statistics. Some kinds of data, however, such as the weights of cars and trucks, may cluster about two or more values, and in such circumstances the meaning of central tendency becomes unclear. This may happen in ratio studies when two or more classes of property are combined.

Class. A set of items defined by common characteristics. (1) In property taxation, property classes such as residen-

tial, agricultural, and industrial may be defined. (2) In assessment, building classification systems based on type of building design, quality of contruction, or structural type are common. (3) In statistics, a predefined category into which data may be put for further analysis. For example, ratios may be grouped into the following classes: less than 0.500, 0.500 to 0.599, 0.600 to 0.699, and so forth.

COD. See **coefficient of dispersion.**

Coefficient of concentration. The percentage of observations falling within a specified percentage (say 15 percent) of a measure of central tendency.

Coefficient of dispersion (COD). The average deviation of a group of numbers from the median expressed as a percentage of the median. In ratio studies, the average percentage deviation from the median ratio.

Coefficient of variation (COV). A standard statistical measure of the relative dispersion of the sample data about the mean of the data; the standard deviation expressed as a percentage of the mean.

Confidence interval. An estimated range of values which is expected to include the true population parameter (mean, median, COD) at a specified confidence level. It can be thought of as a measure of precision for the sample statistic or point estimate.

Confidence level. The degree of probability associated with statistical test or confidence interval, commonly 90, 95, or 99 percent. For example, a 95 percent confidence interval would be expected to include the true population measure (such as the median, mean, or COD) in 95 repeated sampling trials out of 100.

COV. *See* **coefficient of variation.**

Date of sale (date of transfer). The date on which the sale was agreed to. This is considered to be the date the deed, or other instrument of transfer, is signed. The date of recording can be used as a proxy if it is not unduly delayed as it would be in a land contract.

Direct equalization. The process of converting ratio study results into adjustment factors (trends) and changing locally determined appraised or assessed values to more nearly reflect market value or the statutorily required level of assessment. *See also* **equalization** and **indirect equalization.**

Dispersion. The degree to which data are distributed either tightly or loosely around a measure of central tendency. Measures of dispersion include the range, average deviation, standard deviation, coefficient of dispersion, and coefficient of variation.

Distribution-free statistics. A set of robust nonparametric methods whose interpretation or reliability does not depend on stringent assumptions about the distribution of the underlying population from which the sample has been drawn. *See also* **parametric statistics.**

Equalization. The process by which an appropriate governmental body attempts to ensure that property under its jurisdiction is assessed at the same assessment ratio or at the ratio or ratios required by law. Equalization may be undertaken at many different levels. Equalization among use classes (such as agricultural and industrial property) may be undertaken at the local level, as may equalization among properties in a school district and a transportation district; equalization among counties is usually undertaken by the state to ensure that its aid payments are distributed fairly. *See also* direct equalization and indirect equalization.

Fixture. Generally, an asset that has become part of real estate through attachment in such a manner that its removal would result in a loss in value to either the asset or the real estate to which the asset is affixed.

Fractional assessments. Assessments that by law or by practice have assessment ratios different from 1. Usually the assessment ratio is less than 1 and, if assessment biases are present, different classes of property may have different fractional ratios.

Frequency distribution. A table showing the number or percentage of observations falling in the boundaries of a given set of classes. Used in ratio studies to summarize the distribution of the individual ratios. *See also* **class** and **histogram.**

Histogram. A bar chart or graph of a frequency distribution in which the frequencies of the various classes are indicated by horizontal or vertical bars whose lengths are proportional to the number or percentage of observations in each class.

Hypothesis. A statement in inferential statistics the truth of which one is interested in determining.

Independent appraisal. An estimate of value using a model different from that used for assessment purposes. Independent appraisals are used to supplement sales in sales ratio studies or in appraisal ratio studies.

Indirect equalization. The process of computing hypothetical values that represent the oversight agency's best estimate of taxable value, given the statutorily required level of assessment or market value. Indirect equalization ensures proper distribution of intergovernmental transfer payments between state or provincial and local governments despite different levels of appraisal between jurisdictions or property classes. *See also* **equalization** and **direct equalization.**

Interquartile range (interquartile deviation). The result obtained by subtracting the first quartile from the third quartile.

Land contract. An executory contract for the purchase of real property under the terms of which legal title to the property is retained by the vendor until such time as all conditions stated in the contract have been fulfilled; commonly used for installment purchase of real property.

Level of appraisal. The common, or overall, ratio of appraised values to market values. Three concepts are usually of interest: the level required by law, the true or actual level, and the computed level based on a ratio study.

Level of assessment. The common or overall ratio of assessed values to market values. *See also* **level of appraisal.** *Note:* The two terms are sometimes distinguished, but there is no convention determining their meanings when they are. Three concepts are commonly of interest: what the assessment ratio is legally required to be, what the assessment ratio for the population actually is, and what the assessment ratio for the population seems to be, on the basis of a sample and application of inferential statistics. When level of assessment is distinguished from assessment ratio, "level of assessment" usually means either the legal requirement or the true ratio, and "assessment ratio" usually means the true ratio or the sample statistic.

Market value. The most probable price (in terms of money) that a property should bring in a competitive and open market under all conditions requisite to a fair sale, the buyer and seller each acting prudently and knowledgeably, and assuming the price is not affected by undue stimuli. Implicit in this definition is the consummation of a sale as of a specified date and the passing of title from seller to buyer under conditions whereby:

- The buyer and seller are typically motivated.
- Both parties are well informed or well advised, and acting in what they consider their best interests.
- A reasonable time is allowed for exposure in the open market.
- Payment is made in terms of cash or in terms of financial arrangements comparable thereto.
- The price represents the normal consideration for the property sold unaffected by special or creative financing or sales concessions granted by anyone associated with the sale.

Mean. *See* **arithmetic mean.**

Median. A measure of central tendency. The value of the middle item in an uneven number of items arranged or arrayed according to size; the arithmetic average of the two central items in an even number of items similarly arranged.

Median absolute deviation. The median of the absolute deviations from the median. In a symmetrical distribution, the measure approximates one-half the interquartile range.

Median percent deviation. The median of the absolute percent deviations from the median; calculated by dividing the median absolute deviation by one-hundredth of the median.

Nonparametric statistics. *See* **distribution-free statistics.**

Normal distribution. A theoretical distribution often approximated in real world situations. It is symmetrical and bell-shaped; 68 percent of the observations occur within one standard deviation of the mean and 95 percent within two standard deviations of the mean.

Observation. One recording or occurrence of the value of a variable, for example, one sale ratio among a sample of sales ratios.

Outliers. Observations that have unusual values, that is, differ markedly from a measure of central tendency. Some outliers occur naturally; others are due to data errors.

Parameter. Numerical descriptive measure of the population, for example, the arithmetic mean or standard deviation. Parameters are generally unknown and estimated from statistics calculated from a sample of the population.

Parametric statistics. Statistics whose interpretation or reliability depends on the distribution of the underlying data. *See also* **distribution-free statistics.**

Percentile. The values that divide a set of data into specified percentages when the data are arrayed in ascending order. The tenth percentile includes the lowest 10 percent of the values, the twentieth percentile includes the lowest 20 percent of the values, and so forth.

Personal property. *See* **property.**

Points. Prepaid interest on a loan; one point is equal to one percent of the amount of the loan. It is common to deduct points in advance of the loan, so that an individual pays interest on 100 percent of the loan but gets cash on, say, only 99 percent.

Population. All the items of interest, for example, all the properties in a jurisdiction or neighborhood; all the observations in a data set from which a sample may be drawn.

PRD. *See* **price-related differential.**

Price-related differential. The mean divided by the weighted mean. The statistic has a slight bias upward.

Price-related differentials above 1.03 tend to indicate assessment regressivity; price-related differentials below 0.98 tend to indicate assessment progressivity.

Progressivity. *See* **assessment progressivity (regressivity).**

Property. An aggregate of things or rights to things. These rights are protected by law. There are two basic types of property: real and personal. Real property consists of the interests, benefits, and rights inherent in the ownership of land plus anything permanently attached to the land or legally defined as immovable; the bundle of rights with which ownership of real estate is endowed. To the extent that "real estate" commonly includes land and any permanent improvements, the two terms can be understood to have the same meaning. Also called "realty." Personal property is defined as those items that generally are movable or all items not specifically defined as real property. Many states include as personal property the costs associated with placing personal property in service, such as sales tax, freight, and installation. Installation items include, but are not limited to, wiring, foundations, hook-ups, and attachments. Two commonly used tests for distinguishing real and personal property are (1) the intent of the parties and (2) whether the item may be removed from the real estate without damage to either.

Quartiles. The values that divide a set of data into four equal parts when the data are arrayed in ascending order. The first quartile includes the lowest quarter of the data, the second quartile, the second lowest quarter, and so forth.

Random sample. A sample for which each item of the population has an equal chance of being included and, by extension, each possible combination of n items has an equal chance of occurrence.

Range. (1) The maximum value of a sample minus the minimum value. (2) The difference between the maximum and minimum values that a variable may assume.

Ratio study. A study of the relationship between appraised or assessed values and market values. Indicators of market values may be either sales (sales ratio study) or independent "expert" appraisals (appraisal ratio study). Of common interest in ratio studies are the level and uniformity of the appraisals or assessments. *See also* **level of appraisal** and **level of assessment.**

Real property. *See* **property.**

Regressivity. *See* **assessment regressivity.**

Regressivity index. *See* **price-related differential.**

Reliability. Ratio studies typically are based on samples. Statistics derived from these samples may be more or less likely to reflect the true condition in the population depending on the precision or reliability of the sample. Representativeness, sample size, and sample uniformity all contribute to reliability.

Representative sample. A sample of observations from a larger population of observations, such that statistics calculated from the sample can be expected to represent the characteristics of the population being studied.

Sale price. (1) The actual amount of money exchanged for a unit of goods or services, whether or not established in a free and open market. An indicator of market value. (2) Loosely used synonymously with "offering" or "asked" price. *Note:* The sale price is the "selling price" to the vendor and the "cost price" to the vendee.

Sale ratio. The ratio of an appraisal (or assessed) value to the sale price or adjusted sale price of a property.

Sales chasing. Sales chasing is the practice of using the sale of a property to trigger a reappraisal of that property at or near the selling price. Sales chasing causes invalid uniformity results in a sales ratio study and causes invalid appraisal level results unless similar unsold parcels are reappraised by a method that produces an appraisal level for unsold properties equal to the appraisal level of sold properties.

Sales ratio study. A ratio study that uses sales prices as proxies for market values.

Sample. A set of observations selected from a population. If the sample was randomly selected, basic concepts of probability may be applied.

Scatter diagram or **scatterplot.** A graphic means of depicting the relationship or correlation between two variables by plotting one variable on the horizontal axis and one variable on the vertical axis. Often in ratio studies it is informative to determine how ratios are related to other variables. A variable of interest is plotted on the horizontal axis and ratios are plotted on the vertical axis.

Skewed. The quality of a frequency distribution that makes it asymmetrical. Distributions with longer tails on the right than on the left are said to be skewed to the right or to be positively skewed; distributions with longer tails to the left are said to be skewed to the left or to be negatively skewed.

Standard deviation. The statistic calculated from a set of numbers by subtracting the mean from each value and squaring the remainders, adding together all the squares, dividing by the size of the sample less one, and taking the square root of the result. When the data are normally distributed, one can calculate the percentage of observa-

tions within any number of standard deviations of the mean from normal probability tables. When the data are not normally distributed, the standard deviation is less meaningful and one should proceed cautiously.

Standard error. A measure of the precision of a measure of central tendency; the smaller the standard error, the more reliable the measure of central tendency. Standard errors are used in calculating a confidence interval about the arithmetic mean and the weighted mean.

Statistics. Numerical descriptive data calculated from a sample, for example, the median, mean, or coefficient of dispersion. Statistics are used to estimate corresponding measures, termed parameters, for the population.

Stratify. To divide, for purposes of analysis, a sample of observations into two or more subsets according to some criterion or set of criteria.

Stratum, strata (pl.). A class or subset that results from stratification.

Time-adjusted sale price. The price at which a property sold adjusted for the effects of price changes reflected in the market between the date of sale and the date of analysis.

Trimmed mean. The arithmetic mean of a data set identified by the proportion that is trimmed from each end of the ordered array. For example, a 10 percent trimmed mean of a sample of size ten is the average of the eight observations remaining after the largest and smallest observations have been removed.

Value. (1) The relationship between an object desired and a potential owner; the characteristics of scarcity, utility, desirability, and transferability must be present for value to exist. (2) Value may also be described as the present worth of future benefits arising from the ownership of real or personal property. (3) The estimate sought in a valuation. (4) Any number between positive infinity and negative infinity.

Variable. An item of observation that can assume various values, for example, square feet, sales prices, or sales ratios. Variables are commonly described using measures of central tendency and dispersion.

Weighted mean; weighted average. An average in which each value is adjusted by a factor reflecting its relative importance in the whole before the values are summed and divided by their number.

Weighted mean ratio. Sum of the appraised values divided by the sum of the sales prices (or independent estimates of market value), which weights each ratio in proportion to the sale price (or independent estimate of market value).

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Additional readings on ratio studies may be found at MemberLink, IAAO's bibliographic database, http://www.iaao.org. Many websites offer good information on statistics. Because website addresses change frequently, we have not tried to list them here.

SALES VALIDATION QUESTIONNAIRE

Parcel Identification Number _____ Instrument Number _____ - _____

Instrument Type _____ ☐Multi Parcel Sale ☐Split Sale ☐Recording Date _____

Seller (Grantor)
Name _____
Mailing _____
City/St/Zip _____
Phone No. _____

Buyer (Grantee)
Name _____
Mailing _____
City/St/Zip _____
Phone No. _____

Brief Legal Description

_____
_____
_____
_____

Property / Situs Address:
Name and Mailing Address for Tax Statements

_____
_____
_____

PLEASE ANSWER THE FOLLOWING QUESTIONS:

1. Special factors:
   - ☐Sale between immediate family members: SPECIFY THE RELATIONSHIP _____
   - ☐Sale involved corporate affiliates belonging to the same parent company
   - ☐Sale of convenience (correct defects in title; create a joint or common tenancy, etc.)
   - ☐Auction Sale
   - ☐Deed transfer in lieu of foreclosure or repossession
   - ☐Forced sale or sheriff's sale
   - ☐Sale by judicial order (guardian, executor, conservator)
   - ☐Sale involved a government agency or public utility
   - ☐Buyer (new owner) is a religious, charitable, or benevolent organization, school or educational association
   - ☐Land contract or contract for deed
   - ☐Sale of only a partial interest in the real estate
   - ☐Sale involved a trade or exchange of properties
   - ☐**NONE OF THE ABOVE**
2. Check use of property at the time of sale:
   - ☐Single Family Residence ☐Agricultural Land
   - ☐Farm/Ranch with Residence ☐Vacant Lot
   - ☐Condominium Unit ☐Commmercial/Industrial
   - ☐Other: (Specify) _____
3. Was the property rented or leased at the time of sale? ☐Yes ☐No
4. Did the sale price include an existing business? ☐Yes ☐No
5. Was any personal property (such as furniture, equipment, machinery, livestock, crops, business franchise or inventory, etc.) included in the sale price? ☐Yes ☐No
   If yes, please describe _____
   Estimated value of all personal property items included in the sale price $ _____
6. Any recent changes to the property? ☐Yes ☐No
   - ☐New Construction ☐Demolition
   - ☐Remodeling ☐Additions

   Was the work performed by a professional? ☐Yes ☐No
   Date completed _____
   Estimated cost of labor and materials? $ _____
7. Was there a change in use? ☐Yes ☐No
   If yes, please explain _____
8. Does the buyer hold title to any adjoining property? ☐Yes ☐No
9. Was there an appraisal made on the property? ☐Yes ☐No
10. Were any **delinquent** taxes assumed by the purchaser? ☐Yes ☐No
    Amount $ _____
11. Were the **delinquent** taxes included in the sale price? ☐Yes ☐No ☐NA
12. How property was marketed (check all that apply):
    - ☐Listed with real estate agent ☐Displayed a "For Sale" sign
    - ☐Advertised in the newspaper ☐Offered by word of mouth
13. Was the property made available to other potential purchasers? ☐Yes ☐No
    If not, explain _____
14. How long was the property on the market?
15. What was the asking price? _____
16. Date sale price was agreed upon _____
17. Method of financing (check all that apply):
    - ☐New loan(s) from a Financial Institution
    - ☐Name of lending institution: _____
    - ☐Cash down payment $ _____
    - Amount $ _____ Interest rate _____ % Term _____
    - ☐Assumption of existing loan(s)
    - Amount $ _____ Interest rate _____ % Term _____
    - ☐Seller financing
    - Amount $ _____ Interest rate _____ % Term _____
    - ☐Trade of Property: Estimated Value $ _____
    - Describe Traded Property
    - ☐All Cash ☐Not Applicable
18. **Total Sale Price $** _____
19. Was the sale influenced by any unusual circumstances? ☐Yes ☐No
    If yes, please explain: _____
20. Is the total sale price a fair reflection of the market value for the real estate on the sale date? ☐Yes ☐No
    If no, please explain _____

PRINT NAME _____

SIGNATURE _____

☐GRANTOR (SELLER) Daytime phone no. _____
☐GRANTEE (BUYER) Daytime phone no. _____
☐AGENT Daytime phone no. _____

Assessment Standards of the International Association of Assessing Officers

Standard on Certification of Assessing Officers and Valuation Personnel ..... October 1979

Standard on Property Use Codes ..... May 1980

Standard on Assessment Appeal ..... December 1981

Standard on the Application of the Three Approaches to Value in Mass Appraisal ..... September 1983
(revised August 1985)

Standard on Mass Appraisal of Real Property ..... March 1984

Standard on Contracting for Assessing Services ..... September 1986

Standard on Urban Land Valuation ..... July 1987

Standard on Cadastral Maps and Parcel Identifiers ..... January 1988

Standard on Public Relations ..... June 1988

Standard on Facilities, Computers, Equipment, and Supplies ..... January 1989

Standard on Education and Training for Assessing Officers ..... September 1989

Guide to Assessment Administration Standards ..... March 1990

Standard on the Valuation of Property Affected by Environmental Contamination ..... August 1992

Standard on Valuation of Personal Property ..... February 1996

Standard on Facilities, Computers, Equipment, and Supplies ..... May 1996

Standard on Property Tax Policy ..... August 1997

Standard on Ratio Studies ..... July 1999

Single copies of these standards can be obtained at a handling charge of $8.00 per standard from: IAAO, 135 South LaSalle Street, Department 1861, Chicago, IL 60674-1861.

IAAO members can receive these standards at a charge of $5.00 per standard.

A complete set of standards is available for $50.00.